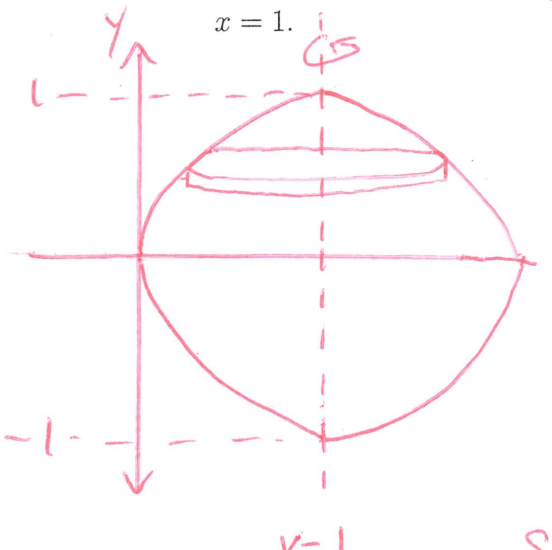
Last Name: KEY   First Name:

MATH 65B - Spring 2018

Groupwork 5: February 20, 2018

1. Find the volume of the region rotated around the line $x = 1$, and bounded by $x = y^2$ and $x = 1$.



Representative ~~slice~~ slice has thickness $\underline{dy}$, and slice is a disk

Radius $R(\cancel{x}y)$ is the distance from the axis of rotation

$\Longrightarrow R(\cancel{x}y) = 1 - y^2$

So then

$$V = \pi \int_a^b R(y)^2\,dy = \pi \int_{-1}^{1} (1-y^2)^2\,dy$$

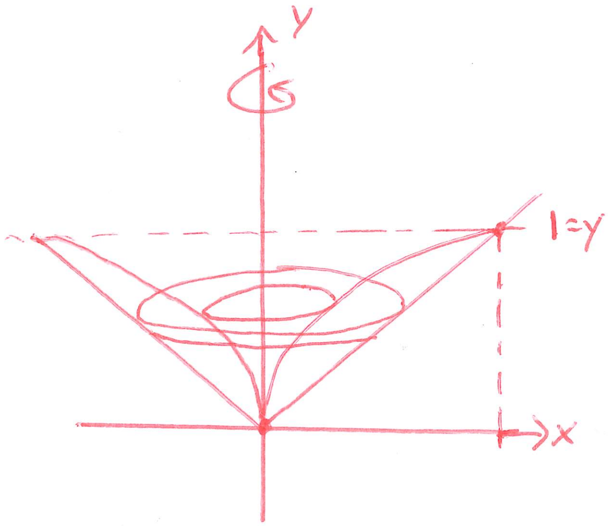
(symmetry) $$= 2\pi \int_0^1 (y^4 - 2y^2 + 1)\,dy$$

$$= 2\pi \left[\frac{y^5}{5} - \frac{2}{3}y^3 + y \right]\Big|_0^1$$

$$= \boxed{\frac{16}{15}\pi}$$

Please, show all work.

2. Find the volume of the region rotated around the y-axis, and bounded by $y = x$ and $y = \sqrt{x}$.



Representative slice is a washer, with thickness of dy.

$R(y) = $ big radius $= y$

$r(y) = $ small radius $= y^2$ $\quad (y = \sqrt{x} \Rightarrow x = y^2)$

$a = 0, b = 1$

$$V = \pi \int_a^b \left(R(y)^2 - r(y)^2 \right) dy$$

$$= \pi \int_0^1 \left((y)^2 - (y^2)^2 \right) dy$$

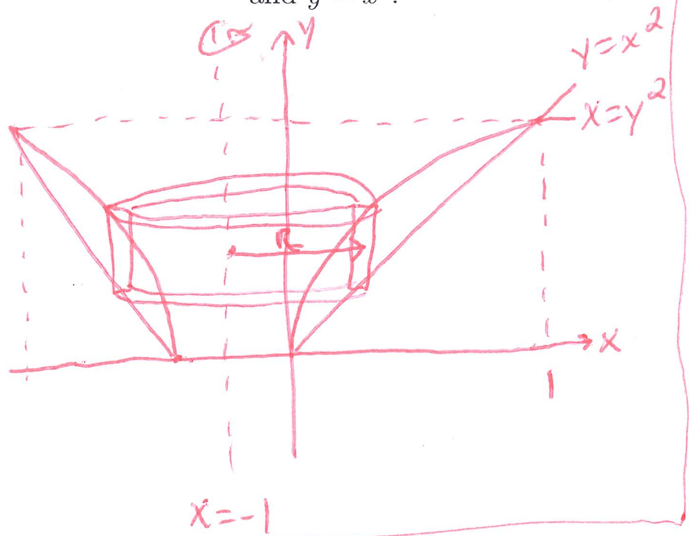
$$= \pi \left(\frac{y^3}{3} - \frac{y^5}{5} \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{3} - \frac{1}{5} \right)$$

$$= \boxed{\frac{2}{15} \pi}$$

Please, show all work.

3. Find the volume of the region rotated around the line $x = -1$, and bounded by $x = y^2$ and $y = x^2$.



Use shell method, with thickness dx

$$V = 2\pi \int_a^b (x+c)(f(x) - g(x))\, dx$$

Here, $f(x) = \sqrt{x}$ $\left(x = y^2 \Rightarrow y = \sqrt{x} \text{ Since we use top of } x = y^2 \right)$

$g(x) = x^2$

$$\Rightarrow V = 2\pi \int_0^1 (x+1)\left[\sqrt{x} - x^2\right] dx$$

$$\Rightarrow V = 2\pi \int_0^1 \sqrt{x}(x+1)\, dx - 2\pi \int_0^1 (x+1)x^2\, dx$$

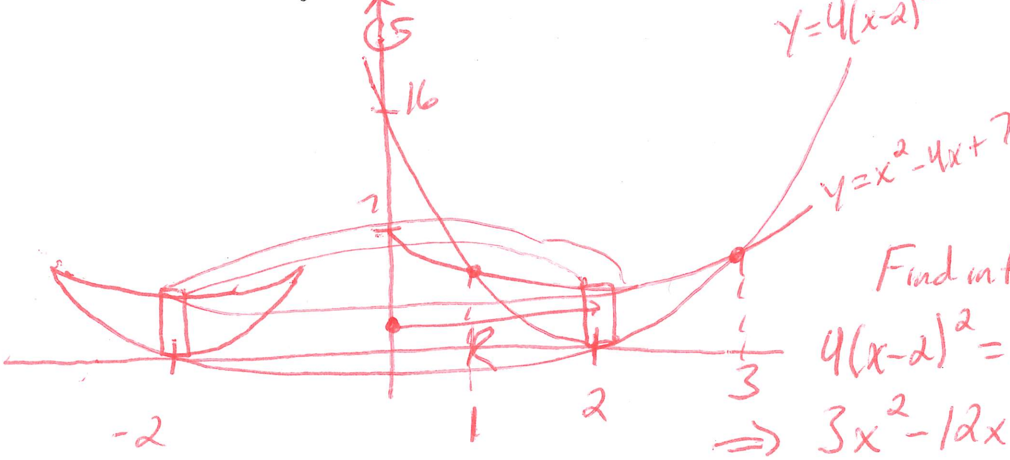
$$= 2\pi \int_0^1 (x^{3/2} + x^{1/2})\, dx - 2\pi \int_0^1 (x^3 + x^2)\, dx$$

$$= 2\pi \left[\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} - \frac{x^4}{4} - \frac{x^3}{3} \right]\Bigg|_0^1$$

$$= 2\pi \left(\frac{2}{5} + \frac{2}{3} - \frac{1}{4} - \frac{1}{3} \right) = \boxed{\frac{29}{30}\pi}$$

Please, show all work.

4. Find the volume of the region rotated around the line y-axis, and bounded by $y = 4(x-2)^2$ and $y = x^2 - 4x + 7$.



Find intersection points first

$4(x-2)^2 = x^2-4x+7$

$\Rightarrow 3x^2-12x+9=0$

$3(x-1)(x-3)=0 \Rightarrow x=1$
$x=3$

$V = 2\pi \int_a^b x\left[f(x)-g(x)\right] dx$ here, $f(x)=x^2-4x+7$
$g(x)=4(x-2)^2$

$V = 2\pi \int_1^3 x\left[x^2-4x+7-4(x-2)^2\right] dx$

$= 2\pi \int_1^3 \left(-3x^3+12x^2-9x\right) dx$

$= -6\pi \int_1^3 \left(x^3-4x^2+3x\right) dx$

$= -6\pi \left[\frac{x^4}{4} - \frac{4}{3}x^3 + \frac{3}{2}x^2\right]\Big|_1^3$

$= -6\pi\left(-\frac{8}{3}\right) = \boxed{16\pi}$

Please, show all work.