LAST NAME: KEY FIRST NAME:

MATH 65B - Spring 2018

Groupwork 6: March 1, 2018

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1. Find the arc length of the curve $y = \ln(\sec(x))$, for $0 \leq x \leq \frac{\pi}{4}$.

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$y = \ln(\sec(x)) \Rightarrow \frac{dy}{dx} = \frac{\sec(x)\tan(x)}{\sec(x)} = \tan(x)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \tan^2(x) \Rightarrow 1+\left(\frac{dy}{dx}\right)^2 = 1+\tan^2(x) = \sec^2(x)$$

$$\Rightarrow L = \int_0^{\pi/4} \sqrt{\sec^2(x)}\,dx = \int_0^{\pi/4} \sec(x)\,dx$$

$$= \ln(\sec(x)+\tan(x))\Big|_0^{\pi/4}$$

$$= \ln(\sqrt{2}+1) - \ln(1+0)$$

$$= \boxed{\ln(\sqrt{2}+1)}$$

Please, show all work.

2. Find the surface area of revolution by rotating the function $y = \sqrt{1+4x}$ about the x-axis for $1 \le x \le 5$.

$$S = 2\pi \int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1+4x}} \Rightarrow 1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{4}{1+4x} = \frac{5+4x}{1+4x}$$

$$\Rightarrow S = 2\pi\int_1^5 \sqrt{1+4x}\cdot\sqrt{\frac{5+4x}{1+4x}}\,dx$$

$$= 2\pi\int_1^5 \sqrt{5+4x}\,dx \qquad u = 5+4x,\ du = 4dx \Rightarrow \frac{1}{4}du = dx$$

$$= 2\pi\int_9^{25} \sqrt{u}\cdot\frac{1}{4}du = \frac{\pi}{2}\int_9^{25} u^{1/2}\,du$$

$$= \frac{\pi}{2}\left(\frac{2}{3}u^{3/2}\right)\Big|_9^{25} = \frac{\pi}{3}(125-27)$$

$$= \boxed{\frac{98}{3}\pi}$$

Please, show all work.

3. Find the surface area of revolution by rotating the function $x = \sqrt{1-y^2}$ about the y-axis for $0 \le y \le \frac{1}{2}$.

$$S = 2\pi \int_a^b x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$\frac{dx}{dy} = \frac{-y}{\sqrt{1-y^2}} \Rightarrow 1+\left(\frac{dx}{dy}\right)^2 = \frac{1}{1-y^2}$$

$$\Rightarrow S = 2\pi \int_0^{1/2} \sqrt{1-y^2} \cdot \frac{1}{\sqrt{1-y^2}}\,dy$$

$$= 2\pi \int_0^{1/2} dy = 2\pi y \Big|_0^{1/2}$$

$$= \boxed{\pi}$$

Please, show all work.

4. A spring has natural length 20 cm. Compare the work W_1 done stretching the spring from 20 cm to 30 cm, with the work W_2 done stretching it from 30 cm to 40 cm. How are W_1 and W_2 related?

$F(x) = Kx$

In SI units 20cm to 30cm ⟺ 0.2m to 0.3m is a 0.1m stretch from equilibrium

Strech from 30cm to 40cm ⟺ 0.3m to 0.4m is a 0.1m stretch from 0.1m from equilibrium to 0.2m from equilibrium

$$\Rightarrow W_1 = \int_0^{0.1} Kx\,dx = \frac{Kx^2}{2}\Big|_0^{0.1} = \frac{1}{200}K$$

$$W_2 = \int_{0.1}^{0.2} Kx\,dx = \frac{Kx^2}{2}\Big|_{0.1}^{0.2} = \frac{3}{200}K$$

$$\Rightarrow \frac{W_2}{W_1} = \frac{\frac{3}{200}K}{\frac{1}{200}K} = 3 \Rightarrow \boxed{W_2 = 3W_1}$$
