Last Name: First Name:

MATH 65B - Spring 2018

Groupwork 7: March 22, 2018

1. Eliminate the parameter for the following parameterized curve. Sketch the curve and use arrows to denote the direction.

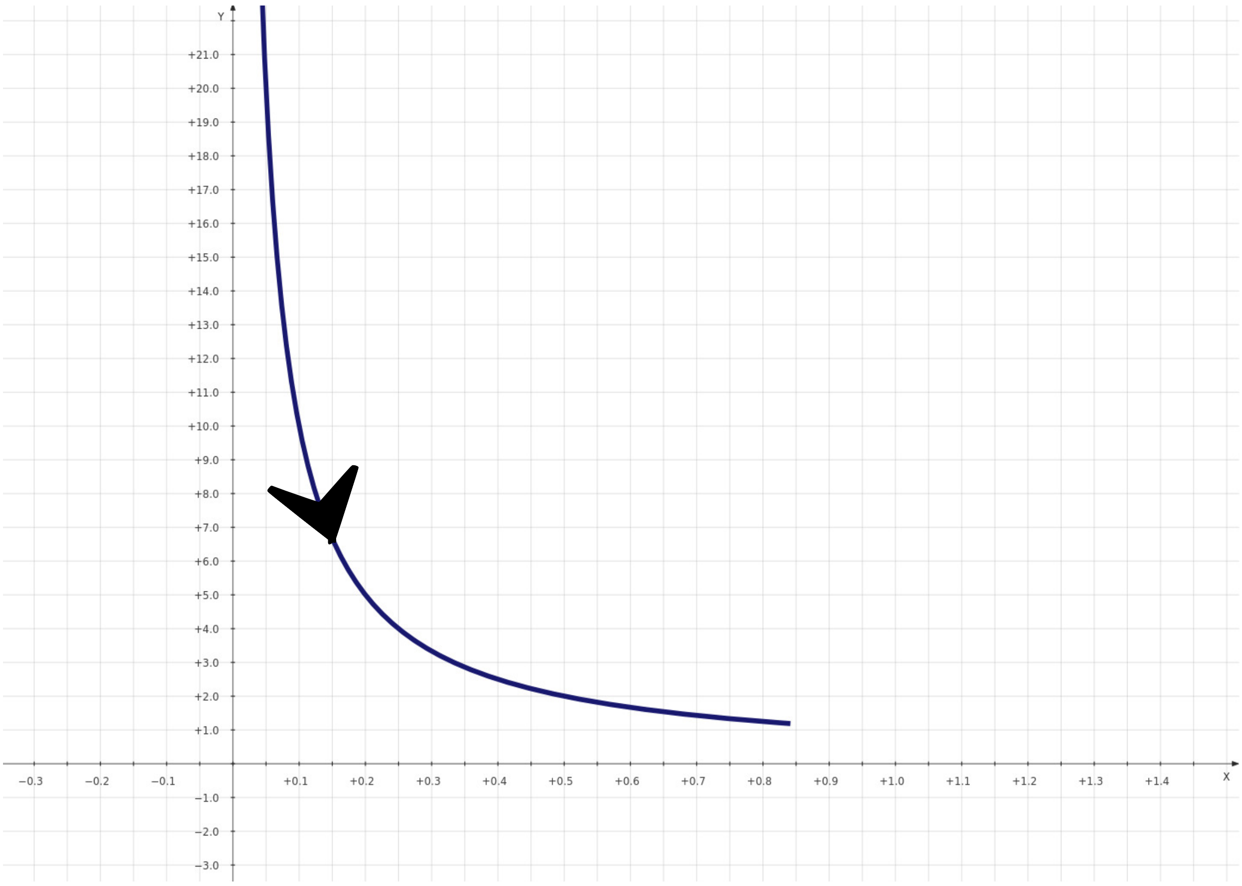
$$x = \sin(t), \quad y = \csc(t), \quad 0 < t < \frac{\pi}{2}$$

Solution:

Notice that we can eliminate the parameter t so that y can be written in terms of x:

$$y = \csc(t) = \frac{1}{\sin(t)} = \frac{1}{x}$$

The restriction of $0 < t < \pi/2$ gives the restrictions: $0 < x < 1$ and $y > 1$, since $x = \sin(t)$ and $y = \csc(t)$. So then the graph is just the function $y = \frac{1}{x}$ with the above restrictions. Note that graph stops as the graph is restricted.



Please, show all work.

2. Eliminate the parameter for the following parameterized curve. Sketch the curve and use arrows to denote the direction.

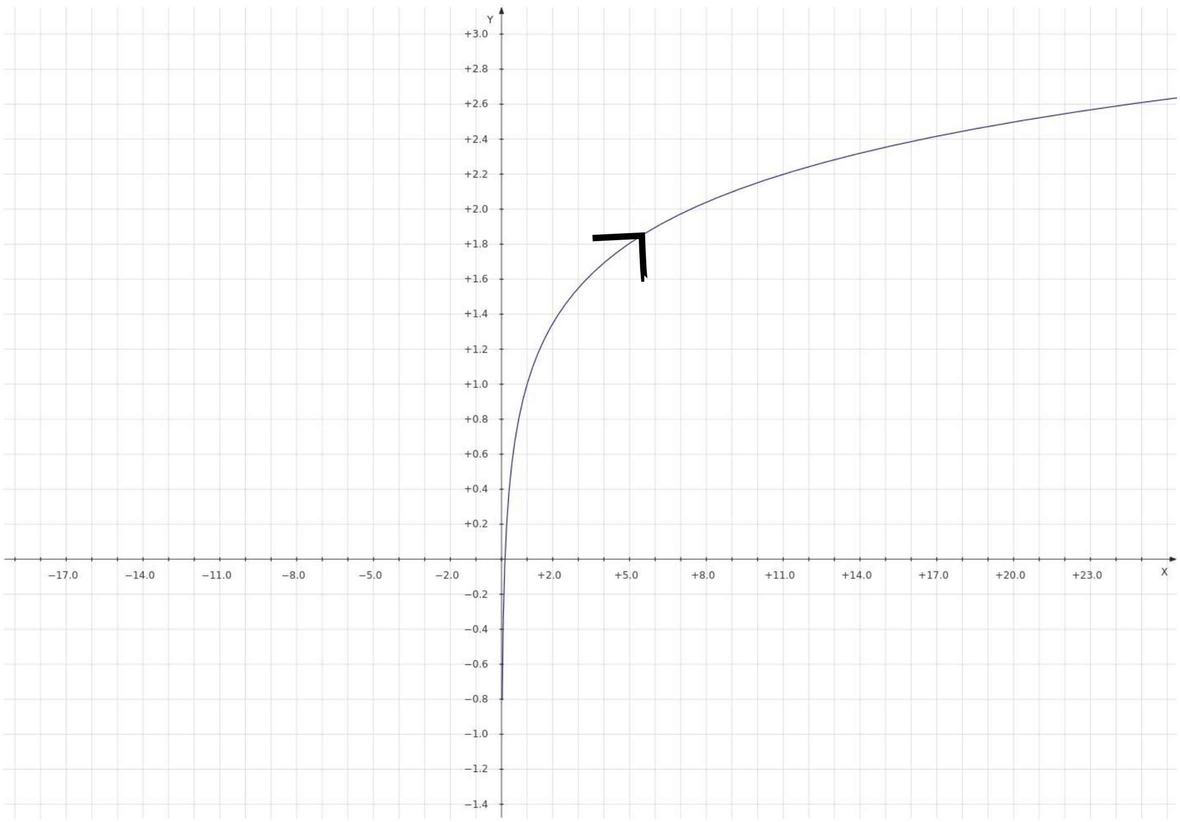
$$x = e^{2t}, \quad y = t + 1, \quad \text{for } -\infty < t < \infty$$

Solution:

Notice that we can eliminate the parameter t so that y can be written in terms of x:

$$\begin{aligned} y = t + 1 \ &\Rightarrow\ t = y - 1 \\ x = e^{2t} \ &\Rightarrow\ x = e^{2(y-1)} \\ &\Rightarrow\ \ln(x) = 2(y-1) \\ &\Rightarrow\ y = 1 + \frac{1}{2}\ln(x) \end{aligned}$$

Since we are plotting over $-\infty < t < \infty$ for the parameter t, the graph will not be restricted as in the previous problem. Now notice that for $t = 0$ we are at the coordinate $(1, 1)$, and for $t = 1$ we are at $(e^2, 2)$. Therefore, we are moving in the direction of positive x and positive y. This gets us the arrow in the plot below.



Please, show all work.

3. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For what values of t is the curve concave up?

$$x = 2\sin(t), \quad y = 3\cos(t), \quad 0 < t < 2\pi$$

<u>Solution</u>:

By direct calculation

$$\begin{aligned}
\frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3\sin(t)}{2\cos(t)} = -\frac{3}{2}\tan(t) \\
\frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{-\frac{3}{2}\sec^2(t)}{2\cos(t)} = -\frac{3}{4}\sec^3(t)
\end{aligned}$$

The curve is concave up where

$$-\sec^3(t) > 0 \Rightarrow \sec^3(t) < 0 \Rightarrow \sec(t) < 0 \Rightarrow \frac{1}{\cos(t)} < 0,$$

which is for $\frac{\pi}{2} < t < \frac{3\pi}{2}$.

Please, show all work.

4. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For what values of t is the curve concave up?

$$x = t - e^t, \quad y = t + e^{-t}$$

<u>Solution</u>:

(a) Use the formula for the derivative, and using some clever algebra:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1 - e^{-t}}{1 - e^t} = \frac{1}{1 - e^t}\left(1 - \frac{1}{e^t}\right) = \frac{1}{1 - e^t}\frac{-(1 - e^t)}{e^t} = -e^{-t}$$

(b) Use the formula for the second derivative:

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}(-e^{-t})}{1 - e^t} = \frac{e^{-t}}{1 - e^t}$$

(c) Since $e^{-t} > 0$ for all t, we only need to check the sign of the denominator. So then, $1 - e^t > 0$ when $e^t < 1$. Therefore, the curve is concave upward for $e^t < 1$ (or $t < \ln(1) = 0$).

Please, show all work.