Section 8.3 - Trigonometric Integrals

$\boxed{\text{Strategy for } \int \sin^m(x)\cos^n(x)\,dx}$

(1) For odd powers of cosine, pull out 1 cosine and use ~~switch~~ switch remaining terms to $\cos^2(x) = 1 - \sin^2(x)$ to all sines. So if $n = 2k+1$

$$\int \sin^m(x)\cos^{2k+1}(x)\,dx = \int \sin^m(x)\cos^{2k}(x)\cos(x)\,dx$$
$$= \int \sin^m(x)\left(1-\sin^2(x)\right)^k \cos(x)\,dx$$

Then use u-substitution $u = \sin(x)$, $du = \cos x\,dx$

(2) For odd powers of sine, we do opposites.
Use $\sin^2(x) = 1 - \cos^2(x)$

$$\int \cos^n(x)\sin^{2k+1}(x)\,dx = \int \cos^n(x)\sin^{2k}(x)\sin(x)\,dx$$
$$= \int \cos^n(x)\left(1-\cos^2(x)\right)^k \sin(x)\,dx$$

Then use $u = \cos(x)$, $du = -\sin(x)\,dx$

Note: If both powers are odd, either can be used.

(3) Even for both sine and cosine.

Use identities
$$\sin^2(x) = \tfrac{1}{2}(1-\cos(2x))$$
$$\cos^2(x) = \tfrac{1}{2}(1+\cos(2x))$$
$$\sin(x)\cos(x) = \tfrac{1}{2}\sin(2x)$$
$$\cos^2(2x) = \tfrac{1}{2}(1+\cos(4x))$$

Examples

(1) $\int \cos^3(x)\,dx$

(2) $\int \sin^5(x)\cos^2(x)$

(3) $\int \sin^2(x)\,dx$

(4) $\int \cos^2(x)\,dx$

(5) $\int \sin^4(x)\,dx$

(6) $\int \sin^2(x)\cos^2(x)\,dx$

$\boxed{\text{Strategy for } \int \tan^m(x)\sec^n(x)\,dx}$

(1) If sec has even power, save 1 copy of $\sec(x)$.
then use $\sec^2(x) = 1+\tan^2(x)$ $(n=2k$ even$)$

$$\Rightarrow \int \tan^m(x)\sec^{2k}(x)\,dx = \int \tan^m(x)\left(1+\tan^2(x)\right)^{k-1}\sec^2(x)\,dx$$

Use $u=\tan(x)$, $du=\sec^2(x)\,dx$

(2) If tan is odd, use $\tan^2(x) = \sec^2(x)-1$ and save copy of $\tan(x)\sec(x)$

$$\Rightarrow \int \tan^{2k+1}(x)\sec^n(x)\,dx = \int (\sec^2(x)-1)^k\sec^{n-1}(x)\sec(x)\tan(x)\,dx$$

use $u=\sec(x)$ $du=\sec(x)\tan(x)$

__Note:__ Others scenarios require: identities, IBP, "creative tricks"

Recall]
$$\int \tan(x)\,dx = \ln|\sec(x)| + C$$
$$\int \sec(x)\,dx = \ln|\sec(x)+\tan(x)| + C$$

Examples

1. $\int \tan^3(x)\,dx$
2. $\int \sec^2(x)\tan(x)\,dx$
3. $\int \tan^3(x)\sec(x)\,dx$

Strategy for $\int \sin(mx)\sin(nx)\,dx$, $\int \sin(mx)\cos(nx)\,dx$, $\int \cos(mx)\cos(nx)\,dx$

Use identities:

1. $\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$
2. $\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$
3. $\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

Ex) $\int \sin(4x)\cos(5x)\,dx$

Ex) $\int \cos(\pi x)\cos(4\pi x)\,dx$