Section 8.4 - Trig Substitution

Strategy for Trig Substitution

Expression	Substitution	Identity
$\sqrt{a^2-x^2}$	$x = a\sin\theta \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$1-\sin^2\theta = \cos^2\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1+\tan^2\theta = \sec^2\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta \quad 0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

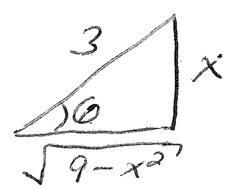
Examples

(1) $\displaystyle\int \frac{\sqrt{9-x^2}}{x^2}\,dx$

$\sqrt{9-x^2} = \sqrt{9-9\sin^2\theta} = \sqrt{9\cos^2\theta} = 3|\cos\theta|$
$= 3\cos\theta$

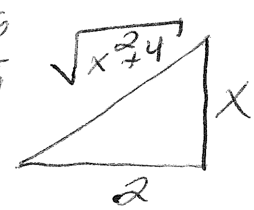
$\cot^2\theta = \csc^2\theta - 1$

$\sin\theta = \frac{x}{3}$



(2) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

$\dfrac{\sec\theta}{\tan^2\theta} = \dfrac{1}{\cos\theta}\cdot\dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{\cos\theta}{\sin^2\theta}$



(3) $\displaystyle\int \frac{x}{\sqrt{3-2x-x^2}}\,dx$

$3-2x-x^2 = 3-(x^2+2x)$
$= 3+1-(x^2+2x+1)$
$= 4-(x+1)^2$

$u = 2\sin\theta$

$= \displaystyle\int \frac{u-1}{\sqrt{4-u^2}}\,du$

$\sqrt{4-u^2} = 2\cos\theta$

(4) $\displaystyle\int \frac{x^3}{(4x^2+9)^{3/2}}\,dx$

$(4x^2+9)^{3/2} = \left(\sqrt{4x^2+9}\right)^3$

$x = \frac{3}{2}\tan\theta$

$\sqrt{4x^2+9} = 3\sec\theta$

$\displaystyle\int \frac{\sin^3\theta}{\cos^2\theta}\,d\theta = \int$ [illegible] $\sin\theta\,d\theta$