Section 10.4 – Comparison Test

Comparison Test: Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.

(a) If $\sum b_n$ converges and $a_n \le b_n \ \forall n$, then $\sum a_n$ converges

(b) If $\sum b_n$ diverges and $a_n \ge b_n \ \forall n$, then $\sum a_n$ diverges

Examples: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n+1}$, $\displaystyle\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$, $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

$\ln(n) > 1$ for $n \ge 3$

Cannot do $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n-1}$ $\quad 2^n > 2^n - 1$

$\Rightarrow \dfrac{1}{2^n-1} > \dfrac{1}{2^n}$

Limit Comparison Test: Suppose $\sum a_n$, $\sum b_n$ are positive termed series

If $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c is a finite number, $c > 0$

then both series converge or both series diverge.

Examples: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n-1}$ compare to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}$

$\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{n^5+5}}$ " " $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2}{n^{5/2}}$ $\left(\text{mult. by } \dfrac{\frac{1}{n^{5/2}}}{\frac{1}{n^{5/2}}}\right)$

$\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ $\quad \dfrac{\ln n}{n^{3/2}} < \dfrac{n^{1/4}}{n^{3/2}} = \dfrac{1}{n^{5/4}}$ $\quad \ln n < n^c$ for any $c > 0$