Section 10.6 - Alternating Series

An alternating series is a series whose terms alternate positive to negative.

Examples: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$

The n^{th} term is of the form $a_n = (-1)^n b_n$ where b_n is positive (or $b_n = |a_n|$

Alternating Series Test: If $\sum_{n=1}^{\infty} (-1)^n b_n$, $b_n > 0$

satisfies (a) $b_{n+1} < b_n \quad \forall n$

(b) $\lim_{n\to\infty} b_n = 0$

then the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^n b_n$ converges.

Examples: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, $\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{4n-1}$

#2) $\lim_{n\to\infty} b_n = \frac{3}{4}$ (b) is not satisfied

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(-1)^n 3n}{4n-1}$ DNE $\Rightarrow$ Test for Divergence

Ex) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$

Strategy for testing Series

How to choose the appropriate test for series

① If a series has a form similar to

$\sum ar^{n-1}$ or $\sum \frac{1}{n^p}$ one of the comparison

tests should be considered.

* For p-test, you choose the highest powers of the numerator and denominator
* If $\sum a_n$ has negative terms, use Comparison test with $\sum |a_n|$

② If $\lim_{n\to\infty} a_n \neq 0$, then Test for Divergence should be used

③ If we have $\sum (-1)^n b_n$, Alternating Series Test is a good choice

④ Ratio test is good for factorials and n^n forms

⑤ Root test is good for $\sum a_n$ that have $(b_n)^n$

⑥ Assuming the hypothesis for Integral test holds, then if $a_n = f(n)$ where $\int_1^\infty f(x)\,dx$ can be done easily, then Integral Test can be used.

Ex) $\sum_{n=1}^{\infty} \frac{n-1}{2n+1}$ ② $\sum_{n=1}^{\infty} n e^{-n^2}$ ⑥ $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ ④

$\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{3n^3+4n^2+2}$ ① $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4+1}$ ③ $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$ ①