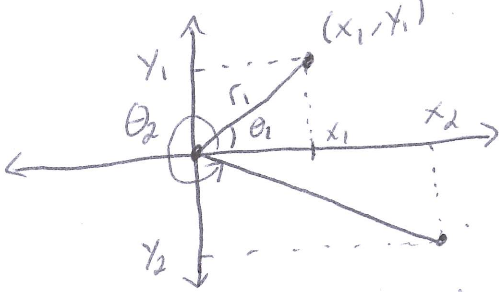
Section 11.3 - Polar Coordinates

Section 11.4 - Graphing Polar Coordinates

In Cartesian coordinates, we use (x,y) coordinates

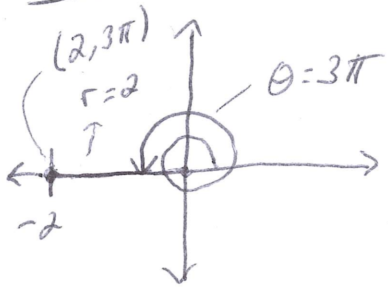
In Polar coordinates, we use (r,θ) where r = "radius" or distance from the origin.

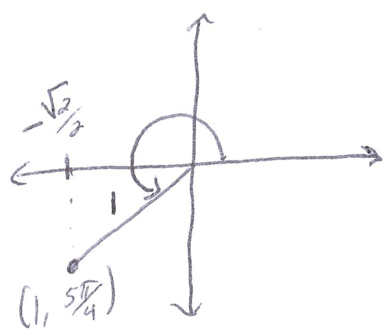
θ = angle of the ray/line segment from origin to point.

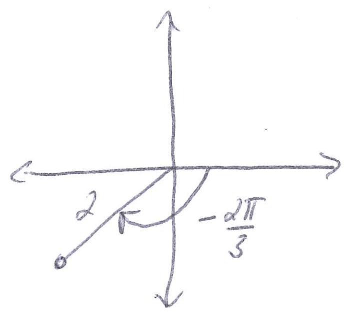


We measure θ from positive x-axis in a counterclockwise manner.
Can be degrees or radians

Examples: Plot $(2,3\pi)$, $(1,\frac{5\pi}{4})$, $(2,-\frac{2\pi}{3})$







Relationship: $\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$ and $\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$

$\Rightarrow x = r\cos\theta$ and $y = r\sin\theta$ (going from (r,θ) to (x,y))

$\Rightarrow r^2 = x^2+y^2$ and $\tan\theta = \frac{y}{x}$ (going from (x,y) to (r,θ))

Convert to Cartesian: $(2,\frac{\pi}{3})$, $(2,3\pi)$, $(1,\frac{5\pi}{4})$, $(2,-\frac{2\pi}{3})$

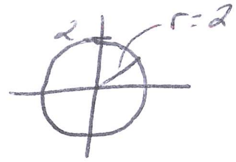
Convert to Polar: $(1,-1)$, $(3\sqrt{3},3)$, $(1,-2)$, $(-1,\sqrt{3})$

In Cartesian coordinates, we can write out curves in multiple ways, like $x=f(y)$, $y=f(x)$ and so on. We can do a similar representation in polar coordinates as $r=f(\theta)$ or $\theta=f(r)$ or more generally $F(r,\theta)=0$ similar to $F(x,y)=0$

Think $x^2+y^2-r^2=0$ is $F(x,y)=0$, $F(x,y)=x^2+y^2-r^2$.

We want to graph functions of these forms.

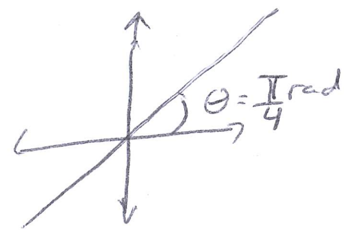
Ex) $r=2 \Rightarrow$ all coordinates look like $(r,\theta)=(2,\theta)$, so for any θ, $r=2$ $\Rightarrow$



we can also deduce that

$r=2 \Rightarrow r^2=4 \Rightarrow x^2+y^2=4$, same graph, a circle.

Ex) $\theta=\pi/4 \Rightarrow$ all coordinates look like $(r,\theta)=(r,\frac{\pi}{4})$, so for any r, the angle is fixed at $\frac{\pi}{4}$ radian. $\Rightarrow$

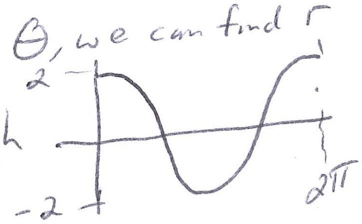


$\theta=\frac{\pi}{4} \Rightarrow \tan\theta=\tan(\frac{\pi}{4}) \Rightarrow \frac{y}{x}=1 \Rightarrow y=x$ line

Ex) $r=2\cos\theta$ For different θ, we can find r

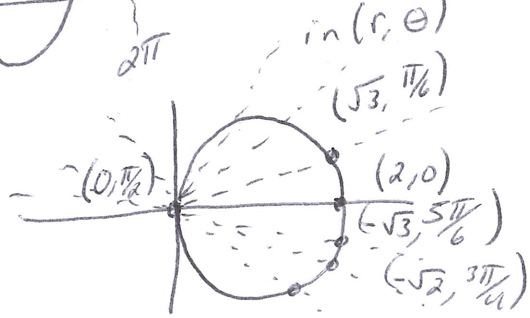
θ	0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
r	2	0	-2	0	2

, graph



or

θ	$\pi/6$	$\frac{\pi}{4}$	$\pi/3$	$\pi/2$	$2\pi/3$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	$\sqrt{3}$	$\sqrt{2}$	1	0	-1	$-\sqrt{2}$	$-\sqrt{3}$	-2



Or see $r=2\cos\theta$

$\Rightarrow \frac{r}{2}=\cos\theta \Rightarrow \frac{r}{2}=\frac{x}{r}$

$\Rightarrow r^2=2x \Rightarrow x^2+y^2=2x$

$x^2-2x+y^2=0$

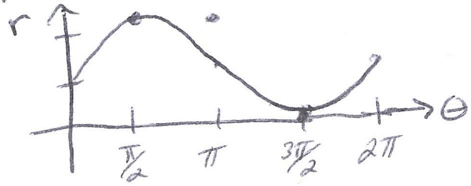
$x^2-2x+1+y^2=1$

$\to (x-1)^2+y^2=1$

circle $r=1$

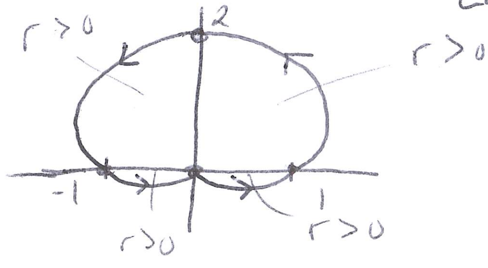
center $(1,0)$

Ex) $r=1+\sin\theta$

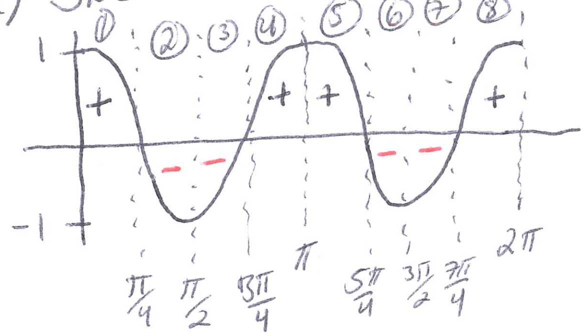


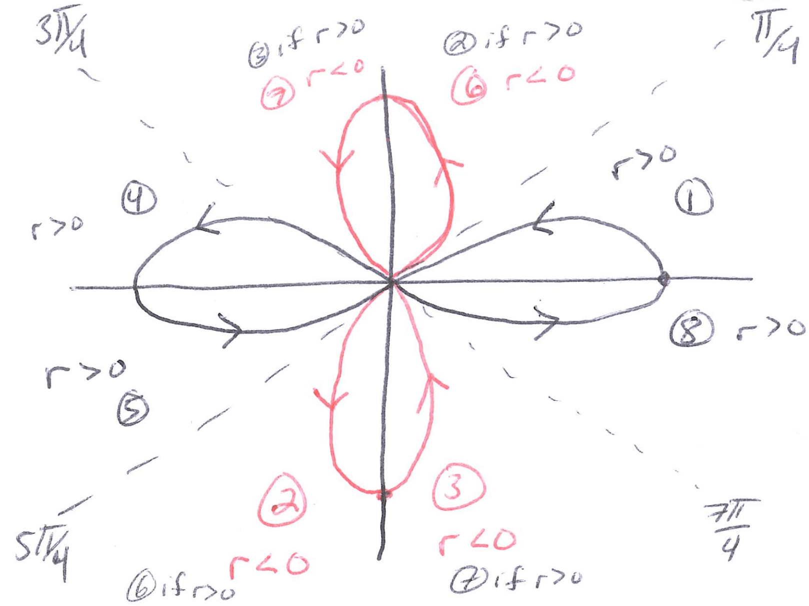
$\Rightarrow$

Cardoid



Ex) Sketch $r = \cos(2\theta)$





4-leaved polar rose

Tangents to polar curves

To find the tangent lines to polar curves, we go back to the parametric equations formula using θ as a parameter.

Let $x = r\cos\theta = f(\theta)\cos\theta$ if $r = f(\theta)$
$y = r\sin\theta = f(\theta)\sin\theta$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{df}{d\theta}\sin\theta + f(\theta)\cos\theta}{\frac{df}{d\theta}\cos\theta - f(\theta)\sin\theta} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

We have horizontal tangents where $\frac{dy}{d\theta} = 0$ and $\frac{dx}{d\theta} \neq 0$
vertical tangents where $\frac{dx}{d\theta} = 0$ and $\frac{dy}{d\theta} \neq 0$
indeterminate if $\frac{dy}{d\theta} = \frac{dx}{d\theta} = 0$ since $\frac{dy}{dx} = \frac{0}{0}$

Ex) Let $r = 1 + \sin\theta$ for $0 \le \theta \le 2\pi$. Find the tangent line at $\theta = \pi/4$. Find values of θ where tangent line is vertical and horizontal.

See PDF.