MATH 65B - Spring 2018

Practice Midterm Answers

1. $-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)+C$

2. $\frac{1}{16}x+\frac{1}{16}\sin(2x)-\frac{1}{64}\sin(4x)-\frac{1}{16}\left(\sin(2x)-\frac{\sin^3(2x)}{3}\right)+C$

3*. $\ln\left|\frac{\sqrt{1+x^2}}{x}-\frac{1}{x}\right|+\sqrt{1+x^2}+C$ or $-\ln\left|\frac{\sqrt{1+x^2}}{x}+\frac{1}{x}\right|+\sqrt{1+x^2}+C$

4. $2\ln|x|+\frac{1}{x}+3\ln|x+2|+C$

5. $-\frac{2}{e}\ \Rightarrow$ convergent

6. $V=\frac{21}{2}\pi$

7. $V=2\pi^2r^2R$

8. $S=3\pi$

9. $W=9$ ft $\cdot$ lbs

***Note:** The two different answers come down to how you do the integral of $\csc(x)$. In my solutions I do the following:

$$\begin{aligned}\int\csc(x)\ dx &= \int\csc(x)\frac{\csc(x)-\cot(x)}{\csc(x)-\cot(x)}\ dx\\ &= \frac{\csc^2(x)-\csc(x)\cot(x)}{\csc(x)-\cot(x)}\ dx \qquad \text{Use: } u=\csc(x)-\cot(x),\\ &= \int\frac{1}{u}\ du \qquad\qquad du=\csc^2(x)-\csc(x)\cot(x)\ dx\\ &= \ln|u|+C=\ln|\csc(x)-\cot(x)|+C\end{aligned}$$

Now, the book says that the integral of $\csc(x)$ is equal to $-\ln|\csc(x)+\cot(x)|$. What is wrong? Try to prove it! First start with my solution above and try to work to the book's formula:

$$\begin{aligned}\ln|\csc(x)-\cot(x)| &= \ln\left|(\csc(x)-\cot(x))\frac{\csc(x)+\cot(x)}{\csc(x)+\cot(x)}\right|\\ &= \ln\left|\frac{\csc^2(x)-\cot^2(x)}{\csc(x)+\cot(x)}\right|=\ln\left|\frac{1}{\csc(x)+\cot(x)}\right|\\ &= \ln\left|(\csc(x)+\cot(x))^{-1}\right|\\ &= -\ln|\csc(x)+\cot(x)|\end{aligned}$$

where we use $\csc^2(x)-\cot^2(x)=1$, so the integrals are the same, up to identity.