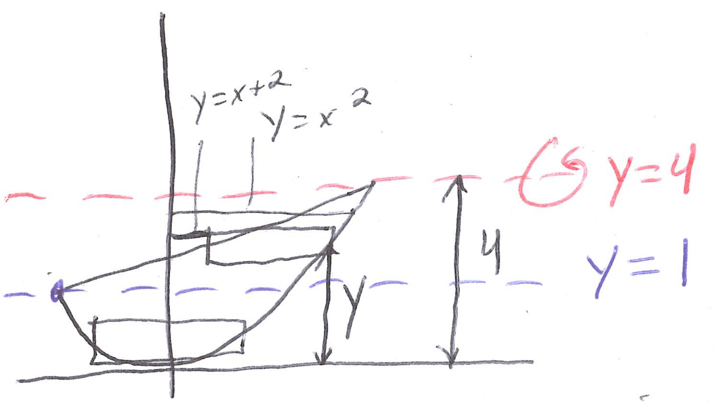
Section 6.2 - #25(d)

Use the shell method to find the volume of the solid generated by rotating the region bounded by

$y = x^2$ and $y = x + 2$ about the line $y = 4$.

Solution: The sketch of the situation is below



$$h_1(y) = \sqrt{y} - (-\sqrt{y})$$

$$h_2(y) = \sqrt{y} - (y-2)$$

$$R(y) = 4 - y$$

from the picture

We need 2 heights since they vary due to the function turning in on itself below $y=1$. We also do (right - left) instead of (top - bottom) for integration wrt y.

The range of integration is thus $0 \le y \le 4$

$$\Rightarrow V = \int_0^1 2\pi R(y) h_1(y)\,dy + \int_1^4 2\pi R(y) h_2(y)\,dy$$

$$= \int_0^1 2\pi (4-y)(\sqrt{y} - (-\sqrt{y}))\,dy + \int_1^4 2\pi (4-y)(\sqrt{y} - (y-2))\,dy$$

$$= 2\pi \int_0^1 (8\sqrt{y} - 2y^{3/2})\,dy + \int_1^4 2\pi (4y^{1/2} - 6y - y\sqrt{y} + y^2 + 8)\,dy$$

$$= 2\pi \left(8 \cdot \tfrac{2}{3} y^{3/2} - 2 \cdot \tfrac{2}{5} y^{5/2}\right)\Big|_0^1$$

$$+ 2\pi \left(4 \cdot \tfrac{2}{3} y^{3/2} - 3y^2 - \tfrac{2}{5} y^{5/2} + \tfrac{y^3}{3} + 8y\right)\Big|_1^4$$

$$= \boxed{\frac{108}{5}\pi}$$