Section 7.5 – #103

Compute the limit $\lim_{x\to 0^+} \dfrac{(\ln(x))^2}{\ln(\sin(x))}$

Solution: $\lim_{x\to 0^+} \dfrac{(\ln(x))^2}{\ln(\sin(x))} = \dfrac{(\ln(0^+))^2}{\ln(\sin(0^+))} = -\dfrac{\infty}{\infty}$ type limit

$$\Rightarrow \lim_{x\to 0^+} \frac{(\ln(x))^2}{\ln(\sin(x))} \overset{L'H}{=\!=} \lim_{x\to 0^+} \frac{2\ln(x)\cdot\frac{1}{x}}{\frac{\cos(x)}{\sin(x)}}$$

$$= \lim_{x\to 0^+} \frac{2\sin(x)}{\frac{x\cos(x)}{\ln(x)}} \quad \left(= \frac{0}{\frac{0\cdot 1}{-\infty}} = -\frac{0}{0} \right)$$

$$\overset{L'H}{=\!=} \lim_{x\to 0^+} \frac{2\cos(x)}{\frac{\ln(x)[\cos(x) - x\sin(x)] - \cos(x)}{(\ln(x))^2}}$$

$$= \lim_{x\to 0^+} \frac{2\cos(x)(\ln(x))^2}{\ln(x)\cos(x)\left[1 - x\tan(x) - \frac{1}{\ln(x)}\right]}$$

$$= \lim_{x\to 0^+} \frac{2\ln(x)}{1 - x\tan(x) - \frac{1}{\ln(x)}} = \frac{-\infty}{1-0-0} = \boxed{-\infty}$$