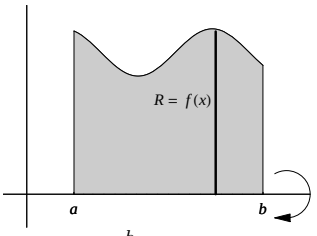
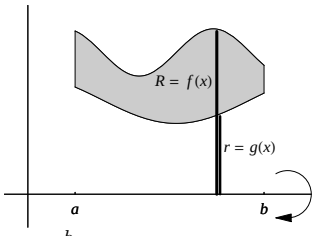
Disk/Washer Method $\int_a^b A(x)\,dx$ or $\int_a^b A(y)\,dy$

Take cross-sections PERPENDICULAR to axis of revolution.
If cross-section is a solid disk, $A = \pi R^2$
If cross-section is a washer/ring/annulus, $A = \pi R^2 - \pi r^2$

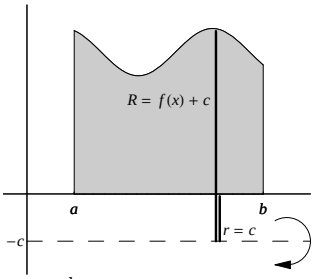
Axis of Revolution is HORIZONTAL: integrate with respect to x:



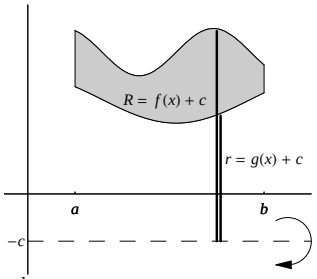
$$V = \int_a^b \pi[f(x)]^2\,dx$$



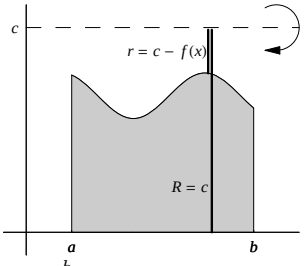
$$V = \int_a^b \pi[f(x)]^2 - \pi[g(x)]^2\,dx$$



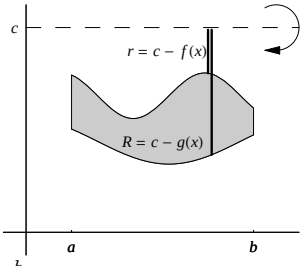
$$V = \int_a^b \pi[f(x)+c]^2 - \pi c^2\,dx$$



$$V = \int_a^b \pi[f(x)+c]^2 - \pi[g(x)+c]^2\,dx$$



$$V = \int_a^b \pi c^2 - \pi[c - f(x)]^2\,dx$$



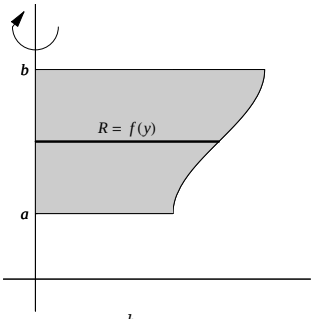
$$V = \int_a^b \pi[c - g(x)]^2 - \pi[c - f(x)]^2\,dx$$

Examples of regions that can be done with either the disk/washer method or the shell method:
see §6.2, #19-30.

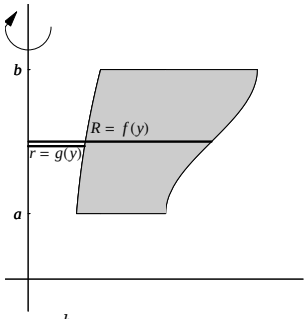
Disk/Washer Method (cont.) $\int_a^b A(x)\,dx$ or $\int_a^b A(y)\,dy$

Take cross-sections PERPENDICULAR to axis of revolution.
If cross-section is a solid disk, $A = \pi R^2$
If cross-section is a washer/ring/annulus, $A = \pi R^2 - \pi r^2$

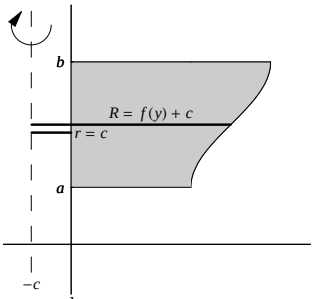
Axis of Revolution is VERTICAL: integrate with respect to y:



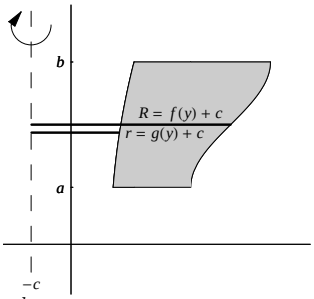
$$V = \int_a^b \pi[f(y)]^2\,dy$$



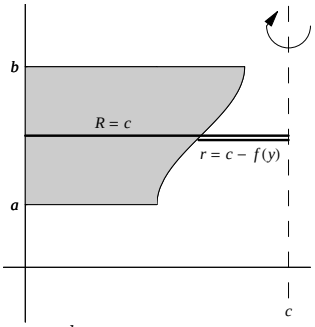
$$V = \int_a^b \pi[f(y)]^2 - \pi[g(y)]^2\,dy$$



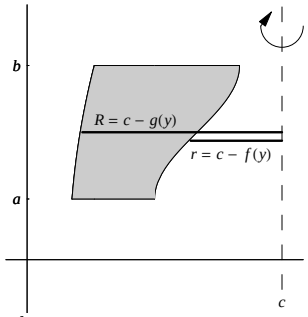
$$V = \int_a^b \pi[f(y)+c]^2 - \pi c^2\,dy$$



$$V = \int_a^b \pi[f(y)+c]^2 - \pi[g(y)+c]^2\,dy$$



$$V = \int_a^b \pi c^2 - \pi[c-f(y)]^2\,dy$$



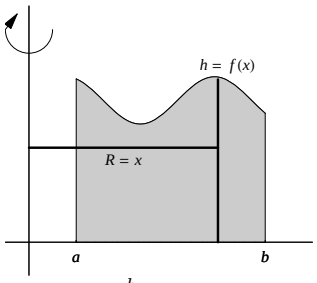
$$V = \int_a^b \pi[c-g(y)]^2 - \pi[c-f(y)]^2\,dy$$

Examples of regions that are best to use the disk/washer method:
$y = 1/x$, $x = 1$, $x = 2$, $y = 0$ about the x-axis, or about the lines $y = -1$, $y = 5$
$y = \cos x$, $y = \sin x$, $x = 0$, $x = \pi/6$ about the x-axis, or about the lines $y = 1$, $y = -1$
$x = 2y - y^2$, $x = 0$, about the y-axis, or about the lines $x = 5$, $x = -5$

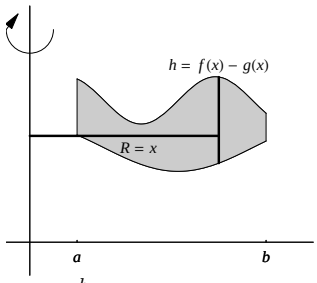
Shell Method $\int_a^b 2\pi Rh\,dx$ or $\int_a^b 2\pi Rh\,dy$

Take cross-sections PARALLEL to axis of revolution.
Figure out the radius R from cross-section to the axis of revolution
Figure out the height h of the cross-section

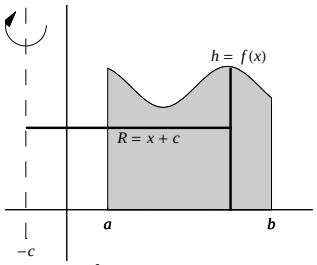
Axis of Revolution is VERTICAL: integrate with respect to x:



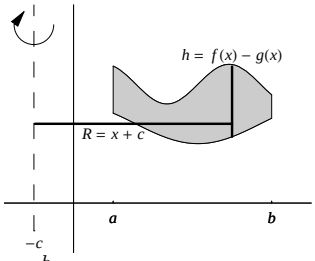
$$V = \int_a^b 2\pi x f(x)\,dx$$



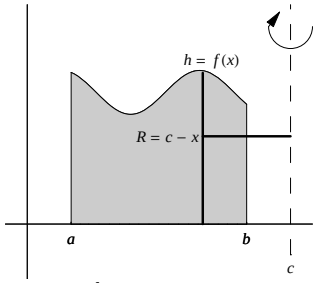
$$V = \int_a^b 2\pi x[f(x) - g(x)]\,dx$$



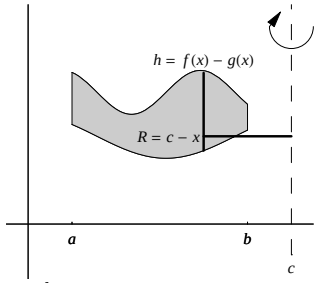
$$V = \int_a^b 2\pi(x + c)f(x)\,dx$$



$$V = \int_a^b 2\pi(x + c)(f(x) - g(x))\,dx$$



$$V = \int_a^b 2\pi(c - x)f(x)\,dx$$



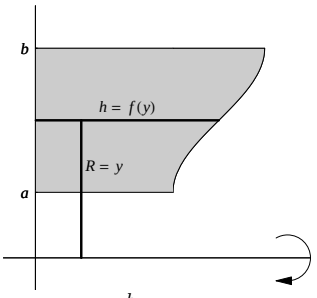
$$V = \int_a^b 2\pi(c - x)(f(x) - g(x))\,dx$$

Examples of regions that are best to use the shell method:
$y = 1/x$, $x = 1$, $x = 2$, $y = 0$ about the y-axis, or about the lines $x = 3$, $x = 0.5$
$y = \cos x$, $y = \sin x$, $x = 0$, $x = \pi/6$ about the y-axis, or about the lines $x = 2$, $x = -2$
$x = 2y - y^2$, $x = 0$, about the x-axis, or about the lines $y = 5$, $y = -5$

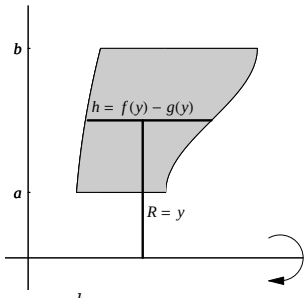
Shell Method (cont.) $\int_a^b 2\pi Rh\,dx$ or $\int_a^b 2\pi Rh\,dy$

Take cross-sections PARALLEL to axis of revolution.
Figure out the radius R from cross-section to the axis of revolution
Figure out the height h of the cross-section

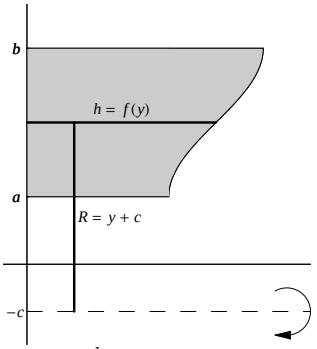
Axis of Revolution is HORIZONTAL: integrate with respect to y:



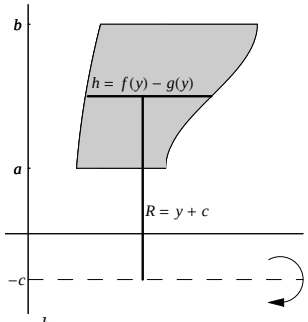
$$V = \int_a^b 2\pi y f(y)\,dy$$



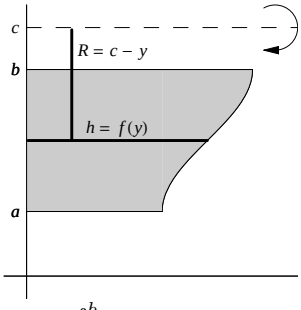
$$V = \int_a^b 2\pi y[f(y) - g(y)]\,dy$$



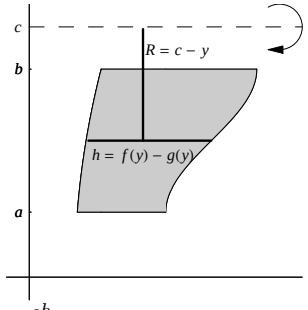
$$V = \int_a^b 2\pi(y + c)f(y)\,dy$$



$$V = \int_a^b 2\pi(y + c)(f(y) - g(y))\,dy$$



$$V = \int_a^b 2\pi(c - y)f(y)\,dy$$



$$V = \int_a^b 2\pi(c - y)(f(y) - g(y))\,dy$$