MATH 25 FINAL December 16, 2015

Name: KEY

Score: ________ / 100

Student ID: ____________________

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	10	11	12	Total
✓													
Score													

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 11 question exam (One extra credit problem can be attempted for a total of 12 questions).
- Students have 2 hours and 30 minutes to complete the exam.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. Clearly box your final answer.
- You **MUST** complete **11** problems for credit. In the above table in the row with the ✓, please mark with a ✓ which problems you want to be graded. If you wish to do a 12^{th} problem for extra credit, please write *EC* in the ✓ row for the problem you wish to be counted for extra credit.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- Each of the 11 questions you choose to do will be graded out of 3 points. The score will then be totaled and multiplied by 3 to get a raw score out of 99 points. One point will be given for clearly writing your name on the exam sheet. This will get you to 100 points. If you choose to do a 12^{th} problem for extra credit, the most that will be awarded for that question will be 3 points. So, the highest possible score on this examination is 103 points out of 100.
- The back of the test can be used for scratch work.

GOOD LUCK!

FORMULAS:

Name	Formula
Foci for ellipse	$c^2 = a^2 - b^2, a > b$
Foci for hyperbola	$c^2 = a^2 + b^2$
n^{th} term of Arithmetic Series	$a_n = a_1 + (n-1)d$
Sum of Arithmetic Series	$S_n = \frac{n}{2}(a_1 + a_n)$
Finite Geometric Series	$\sum_{j=1}^{n} a_1 r^{j-1} = \frac{a_1(1-r^n)}{1-r}$
Infinite Geometric Series	$\sum_{j=1}^{\infty} a_1 r^{j-1} = \frac{a_1}{1-r}$
Binomial coefficients	$\binom{n}{r} = \frac{n!}{r! \cdot (n-r)!}$
Binomial Theorem	$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$

1) Graph the following function. Clearly label all asymptotes and the y-intercept.

$$f(x) = \frac{4}{x^2 - 4} = \frac{4}{(x-2)(x+2)}$$

HA: $y = 0$

VA: $x = \pm 2$

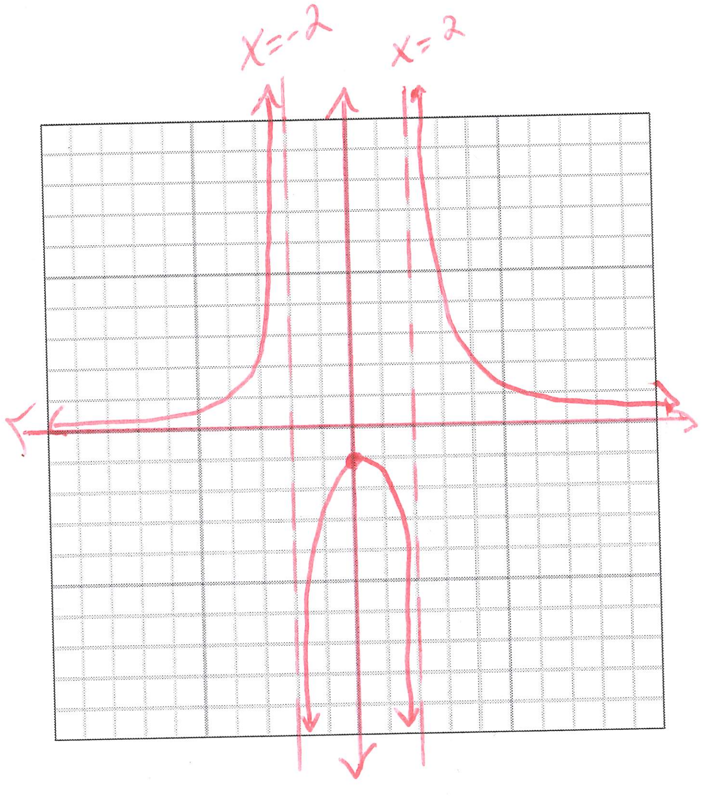
SA: None

No x-ints

yints $\Rightarrow$ $(0,-1)$

Sign chart

+	A	−	A	+
	−2		2	



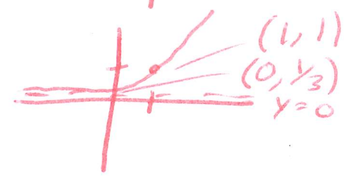
2) Graph the following function. Clearly label all asymptotes and the x and y-intercepts.

$$f(x) = 3^{x-1} - 1$$

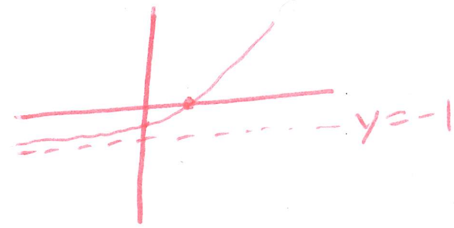
Base Graph: $y = 3^x$

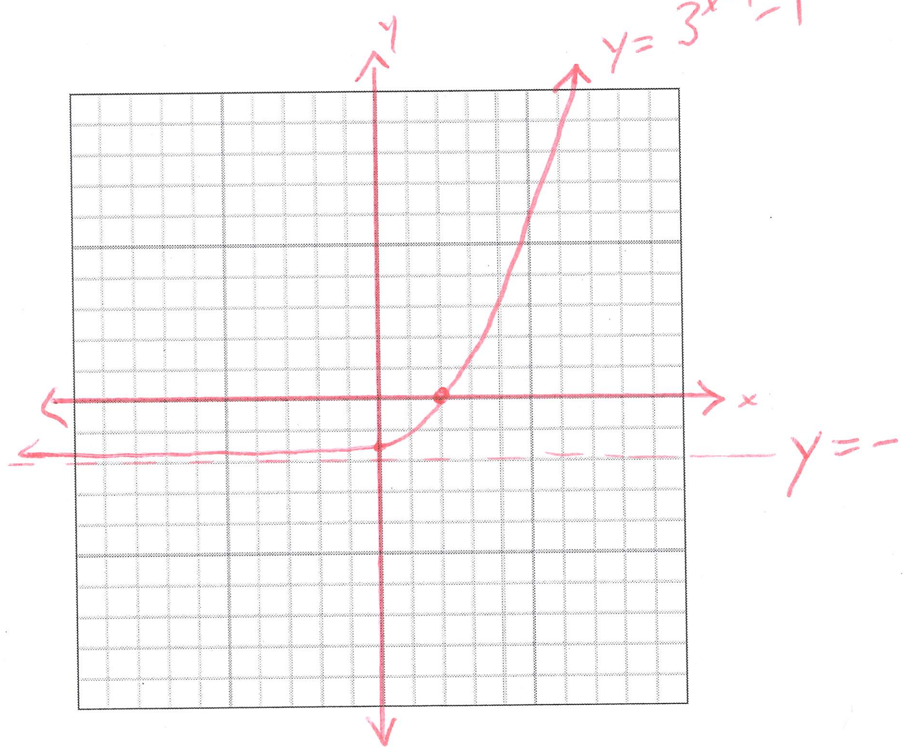
Asymptote @ $y = 0$

Shift right 1 unit $y = 3^{x-1}$



Shift down one unit $y = 3^{x-1} - 1$





3) Solve the following logarithmic equation for x:

$$\log(x+8)+\log(x-1)=1$$

$\Rightarrow \log((x+8)(x-1)) = 1$

$\Rightarrow \log(x^2+8x-x-8) = 1$

$\Rightarrow 10^{\log(x^2+7x-8)} = 10^1$

$\Rightarrow x^2+7x-8 = 10$

$\Rightarrow x^2+7x-18 = 0$

$(x+9)(x-2) = 0$

$x=-9 \quad x=2$

Check: $\left.\begin{array}{l}\log(-9+8)+\log(-9-1)=1 \\ \log(-1)+\log(-10)=1\end{array}\right\} x=-9$

-9 does not work

$\log(2+8)+\log(2-1) = 1$

$\log(10)+\log(1) = 1$

$1+0 = 1$ ✓

$\boxed{x=2}$

4) Solve the following system by Gaussian elimination or Gauss-Jordan elimination:

$$\begin{cases} 4x - 3y &= 6 \\ -\frac{4}{3}x + y &= -2 \end{cases}$$

$$\left[\begin{array}{cc|c} 4 & -3 & 6 \\ -\frac{4}{3} & 1 & -2 \end{array}\right] \xrightarrow{3R_2} \left[\begin{array}{cc|c} 4 & -3 & 6 \\ -4 & 3 & -6 \end{array}\right]$$

$$\xrightarrow{R_1+R_2} \left[\begin{array}{cc|c} 4 & -3 & 6 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow 4x - 3y = 6$$

y is a free variable $\Rightarrow$ $4x = 3y + 6$

$$x = \frac{3}{4}y + \frac{3}{2}$$

So we have $\boxed{\begin{aligned} x &= \frac{3}{4}y + \frac{3}{2} \\ y &= y \end{aligned}}$

or $\boxed{\left\{(x,y) \mid x = \frac{3}{4}y + \frac{3}{2},\ y = y\right\}}$

5) Solve the system of equations using Cramer's Rule:

$$\begin{cases} 2x + y &= 2 \\ 5x + 3y &= 9 \end{cases}$$

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \qquad \begin{array}{ccc} a & b & c \end{array}$$
$$\begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 9 \end{bmatrix}$$

$D = 3(2) - 5(1) = 6 - 5 = 1$

$D_x = \begin{vmatrix} 2 & 1 \\ 9 & 3 \end{vmatrix} = (3)(2) - (9)(1) = 6 - 9 = -3$

$D_y = \begin{vmatrix} 2 & 2 \\ 5 & 9 \end{vmatrix} = (2)(9) - (2)(5) = 18 - 10 = 8$

$x = \dfrac{D_x}{D} = \dfrac{-3}{1} = -3$

$y = \dfrac{D_y}{D} = \dfrac{8}{1} = 8$

Check:

$2(-3) + 8 = -6 + 8 = 2$ ✓

$5(-3) + 3(8) = -15 + 24 = 9$ ✓

$\Rightarrow \boxed{\begin{array}{l} x = -3 \\ y = 8 \end{array}}$

6) Solve the following system of equations using the inverse matrix, A^{-1}.

<u>*Hint*</u>: Find A^{-1} and solve the system $Ax = b$:

$$\begin{cases} x + y & = 1 \\ x + y + z & = 2 \\ \quad\; y + z & = 3 \end{cases}$$

Question 6 on Midterm 2

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

So $Ax = b$

$A^{-1}Ax = A^{-1}b$

$\Rightarrow \quad x = A^{-1}b$

$$x = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \boxed{\begin{aligned} x &= -1 \\ y &= 2 \\ z &= 1 \end{aligned}}$$

7) Write an equation for the ellipse having foci at (0, 1) and (8,1) , a x-vertices at (-1, 1), and (9, 1).

Same as #7 on Practice ~~Midterm~~ Final

See solution on

Practice ~~M~~ Final Solutions

$$\frac{(x-4)^2}{25}+\frac{(y-1)^2}{9}=1$$

8) Write the following equation for the hyperbola in standard form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$. Then identify the values for h, k, a, b.

$$9x^2 - y^2 + 36x + 10y + 2 = 0$$

$$\Rightarrow (9x^2 + 36x) + (-y^2 + 10y) = -2$$

$$9(x^2 + 4x) - (y^2 - 10y) = -2$$

$$9(x^2 + 4x + 4) - (y^2 - 10y + 25) = -2 + 36 - 25$$

$$9(x+2)^2 - (y-5)^2 = 9$$

$$\boxed{\frac{(x+2)^2}{1} - \frac{(y-5)^2}{9} = 1}$$

Center: $(h, k) = (-2, 5)$

$a = 1$

$b = 3$

9) An object in free fall is dropped from a tall cliff. It falls 16 ft. in the 1^{st} second, 48 ft. in the 2^{nd} second, 80 ft. in the 3^{rd} second, and so on.

a) Write the formula for the n^{th} term of an arithmetic sequence that represents the distance in feet that the object will fall in the n^{th} second.
b) How far will the object fall in the 8^{th} second?
c) What is the total distance that the object will fall in 8 seconds?

a) $\{a_j\} = \{16, 48, 80, 112, \ldots\}$

$a_1 = 16,\ d = 32 \Rightarrow a_n = a_1 + (n-1)d$

$\Rightarrow a_n = 16 + (n-1)(32)$

$\Rightarrow \boxed{a_n = 16 + 32(n-1)}$

b) $a_8 = 16 + 32(8-1) = 240$

$\boxed{240 \text{ ft.}}$

c) S_8 is what we need. $S_n = \frac{n}{2}(a_1 + a_n)$

$S_8 = \frac{8}{2}(16 + 240) = 1024$

$\boxed{1024 \text{ ft.}}$

10) Find the sum of the following geometric series: $\sum_{j=0}^{\infty} 4\left(-\frac{3}{4}\right)^{j-1}$.

$j=1$

Using, $\sum_{j=1}^{\infty} a_1 r^{j-1} = \frac{a_1}{1-r}$

$\Rightarrow \sum_{j=1}^{\infty} 4\left(-\frac{3}{4}\right)^{j-1} = \frac{4}{1-(-3/4)} = \frac{4}{\frac{7}{4}} = \boxed{\frac{16}{7}}$

11) Prove that $\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$ for positive integers $n \geq 1$.

Base Case: $n=1$ $\quad 1 \stackrel{?}{=} \frac{1(1+1)}{2} = \frac{2}{2} = 1$ ✓

Assumption Step: True for $n=K$ ⊛

$$\Rightarrow 1+2+3+\dots+K = \frac{K(K+1)}{2}$$

Prove for $n=K+1$: $1+2+3+\dots+K+1 = \frac{(K+1)(K+2)}{2}$

add $(K+1)$ to both sides of ⊛, then

$$1+2+3+\dots+K+(K+1) = \frac{K(K+1)}{2} + (K+1)$$

$$= \frac{K^2+K+2K+2}{2}$$

$$= \frac{K^2+3K+2}{2}$$

$$= \frac{(K+1)(K+2)}{2}$$ ✓

12) a) Use the binomial theorem or Pascal's triangle to expand $(x+y)^3$.

(*Note: You must use the binomial theorem or Pascal's triangle. Expanding as you would do in MATH 410 or 425 will receive no credit.*).

b) Find the 8^{th} term in the expansion of $(2a+b^4)^{10}$.

a)
$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1
\end{array}
$$
$n=3$

$$\Rightarrow \boxed{(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3}$$

b) $\binom{n}{k-1}$ is the binomial coeff for k^{th} term

$n=10$
$k=8$
$k-1=7$

$$\Rightarrow \binom{10}{7} = \frac{10!}{7!\,3!} = \frac{10\cdot\cancel{9}^{3}\cdot\cancel{8}^{4}}{\cancel{3}\cdot\cancel{2}} = 120$$

Let $x=2a$, $y=b^4$

$$\Rightarrow \binom{n}{k-1} x^{n-(k-1)} y^{k-1}$$

$$\Rightarrow 120\, x^3 y^7$$

$$\Rightarrow 120\,(2a)^3 (b^4)^7$$

$$\Rightarrow 120(8)a^3 b^{28}$$

$$\Rightarrow \boxed{960 a^3 b^{28}}$$

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST