

Name: KEY

Score: \_\_\_\_\_ / 100

Student ID: \_\_\_\_\_

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	Total
✓							
Score							

## INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 5 question exam (One extra credit problem can be attempted for a total of 6 questions).
- Students have 50 minutes to complete the exam.
- **PLEASE SHOW ALL WORK.** Any unjustified claims will receive no credit. Clearly box your final answer.
- You **MUST** complete 5 problems for credit. In the above table in the row with the ✓, please mark with a ✓ which problems you want to be graded. If you wish to do a 6<sup>th</sup> problem for extra credit, please write *EC* in the ✓ row for the problem you wish to be counted for extra credit.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- Each of the 5 questions you choose to do will be graded out of 4 points. The score will then be totaled and multiplied by 5 to get a raw score out of 100 points. If you choose to do a 6<sup>th</sup> problem for extra credit, the most that will be awarded for that question will be 3 points. So, the highest possible score on this examination is 103 points out of 100.
- The back of the test can be used for scratch work.

GOOD LUCK!



1) Solve the following system of equations for  $x$ ,  $y$ , and  $z$  using any method.

$$\begin{cases} x - 3y + 3z = -4 \\ 2x + 3y - z = 15 \\ 4x - 3y - z = 19 \end{cases}$$

$$\left[ \begin{array}{ccc|c} 1 & -3 & 3 & -4 \\ 2 & 3 & -1 & 15 \\ 4 & -3 & -1 & 19 \end{array} \right] \xrightarrow[-4R_1+R_3]{-2R_1+R_2} \left[ \begin{array}{ccc|c} 1 & -3 & 3 & -4 \\ 0 & 9 & -7 & 23 \\ 0 & 9 & -13 & 35 \end{array} \right]$$

$$\xrightarrow{-R_2+R_3} \left[ \begin{array}{ccc|c} 1 & -3 & 3 & -4 \\ 0 & 9 & -7 & 23 \\ 0 & 0 & -6 & 12 \end{array} \right] \xrightarrow{-\frac{1}{6}R_3} \left[ \begin{array}{ccc|c} 1 & -3 & 3 & -4 \\ 0 & 9 & -7 & 23 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$\xrightarrow[7R_3+R_2]{-3R_3+R_1} \left[ \begin{array}{ccc|c} 1 & -3 & 0 & 2 \\ 0 & 9 & 0 & 9 \\ 0 & 0 & 1 & -2 \end{array} \right] \xrightarrow{\frac{1}{9}R_2} \left[ \begin{array}{ccc|c} 1 & -3 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$\xrightarrow{3R_2+R_1} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right] \Rightarrow \boxed{\begin{array}{l} x = 5 \\ y = 1 \\ z = -2 \end{array}}$$

Cramer's Rule

$$\Rightarrow D = -54$$

$$D_x = -270$$

$$D_y = -54$$

$$D_z = 108$$

$$x = \frac{D_x}{D} = \frac{-270}{-54} = 5$$

$$y = \frac{D_y}{D} = \frac{-54}{-54} = 1$$

$$z = \frac{D_z}{D} = \frac{108}{-54} = -2$$



2) Solve the following system of equations for  $x$  and  $y$ .

$$\begin{cases} x^2 + 2y^2 = 9 \\ x^2 - 2y^2 = -7 \end{cases}$$

$$\begin{aligned} \textcircled{1} \quad & x^2 + 2y^2 = 9 \\ & + x^2 - 2y^2 = -7 \\ \hline & 2x^2 = 2 \\ & x^2 = 1 \\ & x = \pm 1 \end{aligned}$$

For  $x=1$ ,

$$\begin{aligned} (1)^2 + 2y^2 &= 9 \\ 2y^2 &= 8 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

$$\Rightarrow \boxed{\begin{matrix} (1, 2) \\ (1, -2) \end{matrix}}$$

For  $x=-1$ ,

$$\begin{aligned} (-1)^2 + 2y^2 &= 9 \\ 2y^2 &= 8 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

$$\Rightarrow \boxed{\begin{matrix} (-1, 2) \\ (-1, -2) \end{matrix}}$$

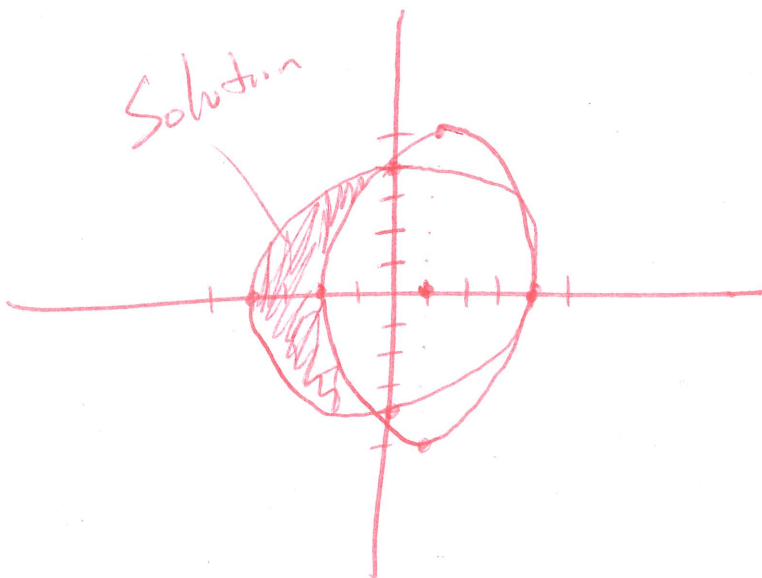


3) Graph the solution set for the following.

$$\begin{cases} x^2 + y^2 \leq 16 \\ \frac{(x-1)^2}{9} + \frac{y^2}{25} \geq 1 \end{cases}$$

$$x^2 + y^2 = 16 \Rightarrow \text{circle radius } 4 \\ \text{center } (0,0)$$

$$\frac{(x-1)^2}{9} + \frac{y^2}{25} = 1 \Rightarrow \text{ellipse } a=3 \\ b=5 \\ \text{Center } (1,0)$$



Test Eqn 1:

$$0 + 0 \leq 16 \checkmark$$

Shade in ~~circle~~ circle

Test Eqn 2:

$$\frac{1}{9} \geq 1 \text{ X}$$

Shade outside ellipse





4) Solve the following system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{cases} x - 3y + z = 4 \\ 2x - 8y + 8z = -2 \\ -6x + 3y - 15z = 9 \end{cases}$$

$$\left[ \begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 2 & -8 & 8 & -2 \\ -6 & 3 & -15 & 9 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -2 & 6 & -10 \\ 0 & -15 & -9 & 33 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & -15 & -9 & 33 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & -54 & 108 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & -3 & 0 & 6 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array} \right] \Rightarrow \boxed{\begin{array}{l} x = 3 \\ y = -1 \\ z = -2 \end{array}}$$



5) Compute  $B \cdot A$  for the following matrices:

$$A = \begin{pmatrix} 1 & 4 & 2 \\ 1 & 3 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$3 \times 2 \cdot 2 \times 3$

$3 \times 3$

$$B \cdot A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 4 & 2 \\ 1 & 3 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 13 & 2 \\ 3 & 11 & 4 \\ 3 & 10 & 2 \end{pmatrix}$$

Incorrect!  $A \cdot B \Rightarrow 2 \times 3 \cdot 3 \times 2 = 2 \times 2$

$$\begin{pmatrix} 1 & 4 & 2 \\ 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 11 & 11 \\ 7 & 6 \end{pmatrix}$$

But not  
what  
question  
asked.



6) Solve the following system of equations using the inverse matrix,  $A^{-1}$ .

$$\begin{cases} x+y & = 1 \\ x+y+z & = 2 \\ y+z & = 3 \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow Ax = b \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$
$$(A^{-1}A)x = A^{-1}b$$
$$x = A^{-1}b$$

$$x = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x = -1 \\ y = 2 \\ z = 1 \end{cases}$$



THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST

