MATH 25 — PRACTICE FINAL — December 16, 2015

Name: KEY

Score: ______ / 100

Student ID: ______

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	10	11	12	Total
✓													
Score													

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 11 question exam (One extra credit problem can be attempted for a total of 12 questions).
- Students have 2 hours and 30 minutes to complete the exam.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. Clearly box your final answer.
- You **MUST** complete **11** problems for credit. In the above table in the row with the ✓, please mark with a ✓ which problems you want to be graded. If you wish to do a 12^{th} problem for extra credit, please write *EC* in the ✓ row for the problem you wish to be counted for extra credit.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- Each of the 11 questions you choose to do will be graded out of 3 points. The score will then be totaled and multiplied by 3 to get a raw score out of 99 points. One point will be given for clearly writing your name on the exam sheet. This will get you to 100 points. If you choose to do a 12^{th} problem for extra credit, the most that will be awarded for that question will be 3 points. So, the highest possible score on this examination is 103 points out of 100.
- The back of the test can be used for scratch work.

GOOD LUCK!

FORMULAS:

Name	Formula
Foci for ellipse	$c^2 = a^2 - b^2, a > b$
Foci for hyperbola	$c^2 = a^2 + b^2$
n^{th} term of Arithmetic Series	$a_n = a_1 + (n-1)d$
Sum of Arithmetic Series	$S_n = \frac{n}{2}(a_1 + a_n)$
Finite Geometric Series	$\sum_{j=1}^{n} a_1 r^{j-1} = \frac{a_1(1-r^n)}{1-r}$
Infinite Geometric Series	$\sum_{j=1}^{\infty} a_1 r^{j-1} = \frac{a_1}{1-r}$
Binomial coefficients	$\binom{n}{r} = \frac{n!}{r!\cdot(n-r)!}$
Binomial Theorem	$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$

1) Graph the following function. Clearly label all asymptotes and the y-intercept.

$$f(x) = \frac{5x}{x^2 - x - 6} = \frac{5x}{(x-3)(x+2)}$$

HA: $y = 0$ since deg(bottom) > deg(top)

VA: $x - 3 = 0 \Rightarrow x = 3$
$x + 2 = 0 \quad\; x = -2$

SA: None

yint is @ where $x = 0 \Rightarrow y = 0$
So $(0,0)$

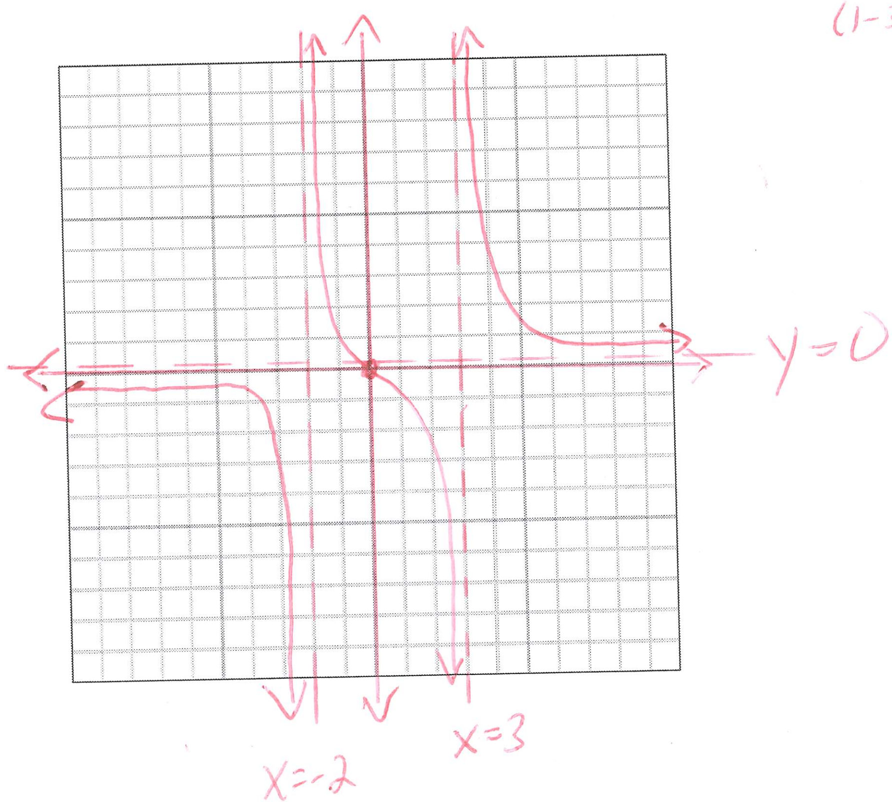
Sign Chart

$-$ A $+$ 0 $-$ $+$ (at -2, 0, 3)

A: asymptote
0 = zero

Ex) for $x = 1$

$$\frac{5(1)}{(1-3)(1+2)} = \frac{+}{(-)(+)} = (-)$$



2) Graph the following function. Clearly label all asymptotes and the x and y-intercepts.

$$f(x) = -4^{x+1} - 2$$

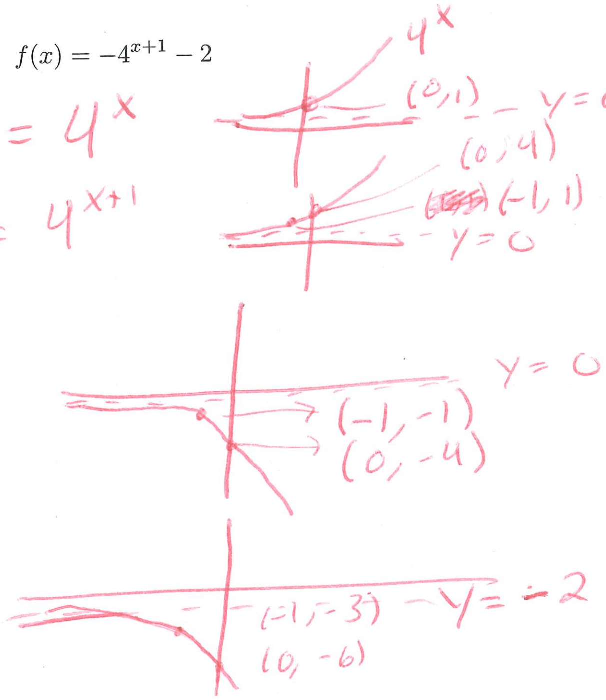
Base Graph : $y = 4^x$

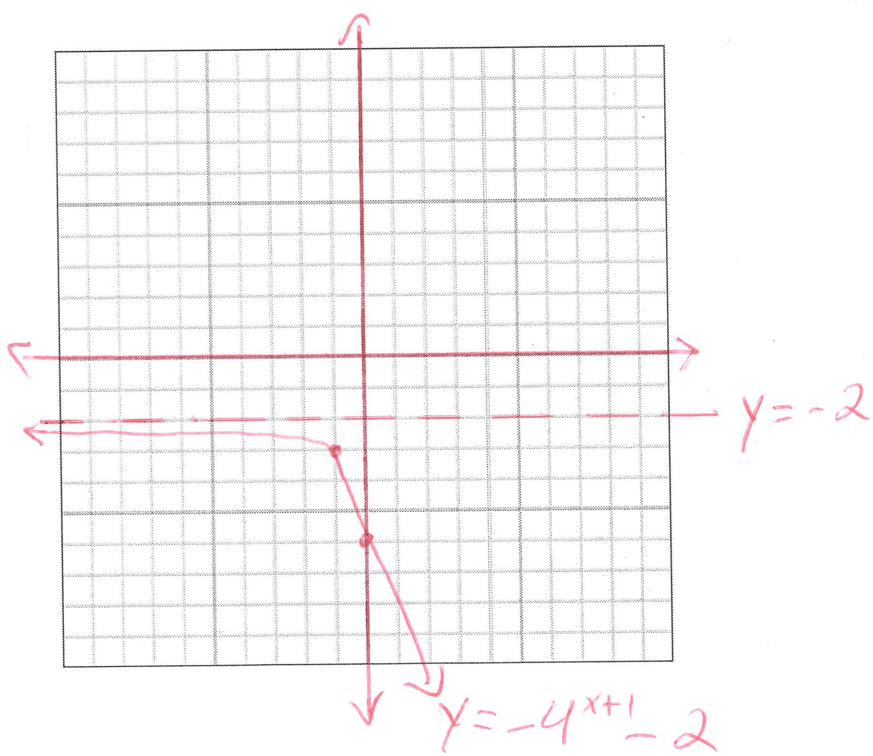
(1) Shift Right 1 unit $y = 4^{x+1}$

(2) Reflect over X-axis

(3) Shift down 2 units

Asymptote shifts by 2





3) Solve the following logarithmic equation for x:

$$\log_3(y) + \log_3(y+6) = 3$$

$\Rightarrow \log_3(y(y+6)) = 3$

$3^{\log_3(y(y+6))} = 3^3$

$y^2 + 6y = 27$

$y^2 + 6y - 27 = 0$

$(y+9)(y-3) = 0$

$y = -9 \quad y = 3$

Check $\log_3(-9) + \log_3(-3) = 3$ X

No "–" in log!

$\log_3(3) + \log_3(9) \stackrel{?}{=} 3$

$1 + 2 = 3$ ✓

$\Rightarrow \boxed{y = 3}$

4) Solve the following system by Gaussian elimination or Gauss-Jordan elimination:

$$\begin{cases} 2(x-y) & = 4x+y-40 \\ 9y & = 105-3x \end{cases}$$

Fix the system to standard form

$$\begin{cases} 2x-2y=4x+y-40 \\ 3x+9y=105 \end{cases}$$

$$\begin{cases} -2x-3y=-40 \\ 3x+9y=105 \end{cases}$$

$$\Rightarrow \left[\begin{array}{cc|c} -2 & -3 & -40 \\ 3 & 9 & 105 \end{array}\right] \xRightarrow{\frac{1}{3}R_2} \left[\begin{array}{cc|c} -2 & -3 & -40 \\ 1 & 3 & 35 \end{array}\right]$$

$$\xRightarrow{R_1+2R_2} \left[\begin{array}{cc|c} -2 & -3 & -40 \\ 0 & 3 & 30 \end{array}\right] \xRightarrow{\frac{1}{3}R_2} \left[\begin{array}{cc|c} -2 & -3 & -40 \\ 0 & 1 & 10 \end{array}\right]$$

or

$$\xRightarrow{R_2+R_1} \left[\begin{array}{cc|c} -2 & 0 & -10 \\ 0 & 3 & 30 \end{array}\right]$$

$$\xRightarrow[\frac{1}{3}R_2]{-\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 10 \end{array}\right]$$

$$\Rightarrow \boxed{\begin{array}{l} x=5 \\ y=10 \end{array}}$$

5) Solve the system of equations using Cramer's Rule:

$$\begin{cases} 3x - 4y &= 9 \\ -5x + 6y &= 2 \end{cases}$$

$$\begin{array}{ccc} a & b & c \end{array}$$
$$\begin{bmatrix} 3 & -4 & 9 \\ -5 & 6 & 2 \end{bmatrix}$$

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} 3 & -4 \\ -5 & 6 \end{vmatrix} = -2$$

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = \begin{vmatrix} 9 & -4 \\ 2 & 6 \end{vmatrix} = 62$$

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3 & 9 \\ -5 & 2 \end{vmatrix} = 51$$

$$X = \frac{D_x}{D} = \frac{-62}{2} = -31$$

$$Y = \frac{D_y}{D} = -\frac{51}{2} = -\frac{51}{2}$$

$$\boxed{\begin{array}{l} x = -31 \\ y = -\frac{51}{2} \end{array}}$$

6) Solve the following system of equations.

$$\begin{cases} -2x+5y-4z &= -4 \\ x-2y+z &= 3 \\ x-5y+9z &= -5 \end{cases}$$

$$\left[\begin{array}{ccc|c} -2 & 5 & -4 & -4 \\ 1 & -2 & 1 & 3 \\ 1 & -5 & 9 & -5 \end{array}\right] \xrightarrow[R_1+2R_2]{R_1+2R_2} \left[\begin{array}{ccc|c} -2 & 5 & -4 & -4 \\ 0 & 1 & -2 & 2 \\ 0 & -5 & 14 & -14 \end{array}\right]$$

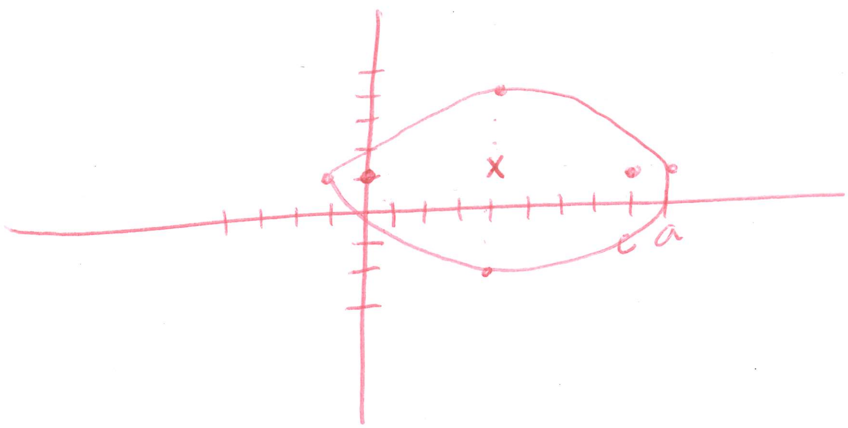
$$\xrightarrow{5R_1+R_2} \left[\begin{array}{ccc|c} -2 & 5 & -4 & -4 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 4 & -4 \end{array}\right] \xrightarrow{\frac{1}{4}R_3} \left[\begin{array}{ccc|c} -2 & 5 & -4 & -4 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow[4R_3+R_1]{2R_3+R_2} \left[\begin{array}{ccc|c} -2 & 5 & 0 & -8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right] \xrightarrow{-5R_2+R_1} \left[\begin{array}{ccc|c} -2 & 0 & 0 & -8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right] \Rightarrow \boxed{\begin{array}{l} X=4 \\ Y=0 \\ Z=-1 \end{array}}$$

7) Write an equation for the ellipse having foci at (0, 1) and (8,1) , a x-vertices at (-1, 1), and (9, 1).

Graph



$c = (0,1)$
$(8,1)$

$a = (-1,1)$
$(9,1)$

$c^2 = a^2 - b^2$

$\downarrow$

$b^2 = a^2 - c^2$

$b^2 = 25 - 16$

$b^2 = 9$

$b = 3$

Center is halfway between vertices and foci

$\Rightarrow$ center $(4,1)$

$\Rightarrow a = 5$
$c = 4$

$$\Rightarrow \boxed{\frac{(x-4)^2}{25} + \frac{(y-1)^2}{9} = 1}$$

8) Write the following equation for the ellipse in standard form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. Then identify the values for h, k, a, b.

$$3x^2 + 2y^2 - 30x - 4y + 59 = 0$$

$$\Rightarrow (3x^2 - 30x) + (2y^2 - 4y) = -59$$

$$\Rightarrow 3(x^2 - 10x) + 2(y^2 - 2y) = -59$$

$$\Rightarrow 3(x^2 - 10x + 25) + 2(y^2 - 2y + 1) = -59 + 75 + 2$$

$$\Rightarrow 3(x-5)^2 + 2(y-1)^2 = 18$$

$$\Rightarrow \frac{(x-5)^2}{6} + \frac{(y-1)^2}{9} = 1$$

$$\boxed{\begin{gathered}(h,k) = (5,1)\\ a = \sqrt{6}\\ b = 3\end{gathered}}$$

9) Consider the sequence $\{a_j\} = \{1, 6, 11, 16, 21, ...\}$.

a) Identify the type of sequence $\{a_j\}$.
b) What is the value of the term a_{100}?
c) Find the sum of the first 50 terms.

a) Arithmetic Sequence

b) $a_n = a_1 + (n-1)d$

$d = 5, a_1 = 1, n = 100, (n-1) = 99$

$a_{100} = a_1 + 99d$

$a_{100} = 1 + 99(5)$

$\boxed{a_{100} = 496}$

c) $a_{50} = 1 + 49(5) = 246$

$\boxed{S_{50} = \frac{50}{2}(1 + 246) = 6175}$

10) Find the sum of the following geometric series: $\sum_{j=0}^{\infty} \left(\frac{2}{3}\right)^{j-1}$.

$j=1$

$$\sum_{j=1}^{\infty} ar^{j-1} \Rightarrow a=1,\ r=\frac{2}{3}$$

$$= \frac{a}{1-r}$$

$$\sum_{j=1}^{\infty} \left(\frac{2}{3}\right)^{j-1} = \frac{a}{1-r} = \frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = \boxed{3}$$

11) Prove that $2 + 6 + 10 + ... + (4n - 2) = 2n^2$ for positive integers $n \geq 1$.

Proof by Induction

Base Case $n=1 \Rightarrow 4n-2 \Rightarrow (4(1)-2) = 2$
$2n^2 \Rightarrow 2(1)^2 = 2$ ✓

Assumption: True for $n=K$

Prove for $n=K+1$

Assumption step $\Rightarrow 2+6+10+\ldots+(4K-2) = 2K^2$

Need to show $2+6+10+\ldots+(4K-2)+(4(K+1)-2) = 2(K+1)^2$

By assumption $2+6+10+\ldots+(4K-2) = 2K^2$

$2+6+10+\ldots+(4K-2)+(4(K+1)-2) = 2K^2+(4(K+1)-2)$
$= 2K^2+4K+4-2$
$= 2K^2+4K+2$
$= 2(K^2+2K+1)$
$= 2(K+1)^2$ ✓

True for $n=K+1$, so true for $n \geq 1$ positive integer n.

12) Use the binomial theorem or Pascal's triangle to expand $(x+y)^5$.

Method 1

Binomial Theorem: $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$

$$\Rightarrow (\cancel{a}+\cancel{b})^5 = \sum_{r=0}^{5} \binom{5}{r} a^{5-r} b^r$$

$$= \binom{5}{0} a^5 b^0 + \binom{5}{1} a^4 b^1 + \binom{5}{2} a^3 b^2 + \binom{5}{3} a^2 b^3 + \binom{5}{4} a^1 b^4 + \binom{5}{5} a^0 b^5$$

$$= 1 \cdot a^5 b^0 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + 1a^0b^5$$

$$\Rightarrow \boxed{(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5} \quad ⊛$$

Method 2: Pascal's Triangle

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
1   5  10  10   5   1
```

Coefficients

Powers of variables add up to n and decrease for one variable, increase in other.

We get the same result as ⊛.

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST