

Name: KEY

Score: _____ / 100

Student ID: _____

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	Total
✓							
Score							

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 5 question exam (One extra credit problem can be attempted for a total of 6 questions).
- Students have 50 minutes to complete the exam.
- **PLEASE SHOW ALL WORK.** Any unjustified claims will receive no credit. Clearly box your final answer.
- You **MUST** complete 5 problems for credit. In the above table in the row with the ✓, please mark with a ✓ which problems you want to be graded. If you wish to do a 6th problem for extra credit, please write *EC* in the ✓ row for the problem you wish to be counted for extra credit.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- Each of the 5 questions you choose to do will be graded out of 4 points. The score will then be totaled and multiplied by 5 to get a raw score out of 100 points. If you choose to do a 6th problem for extra credit, the most that will be awarded for that question will be 3 points. So, the highest possible score on this examination is 103 points out of 100.
- The back of the test can be used for scratch work.

GOOD LUCK!

1) Solve the following system of equations:

$$\begin{cases} x - 2y + 3z = 7 \\ 2x + y + z = 4 \\ -3x + 2y - 2z = -10 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 2 & 1 & 1 & 4 \\ -3 & 2 & -2 & -10 \end{array} \right]$$

$$\begin{array}{l} -2R_1 + R_2 \\ 3R_1 + R_3 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 5 & -5 & -10 \\ 0 & -4 & 7 & 11 \end{array} \right] \xrightarrow{\frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & -1 & -2 \\ 0 & -4 & 7 & 11 \end{array} \right]$$

$$\xrightarrow{4R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 3 & 3 \end{array} \right] \xrightarrow{\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\boxed{\begin{array}{l} x = 2 \\ y = -1 \\ z = 1 \end{array}}$$

2) Solve the following system of equations:

$$\begin{cases} 2x^2 + y^2 = 24 \\ x^2 - y^2 = -12 \end{cases}$$

$$\begin{array}{r} 2x^2 + y^2 = 24 \\ + \quad x^2 - y^2 = -12 \\ \hline 3x^2 = 12 \end{array}$$

$$x^2 = 4$$

$$x = \pm 2$$

For $x = 2$

$$\Rightarrow (2)^2 - y^2 = -12$$

$$4 - y^2 = -12$$

$$16 = y^2$$

$$y = \pm 4$$

$$\Rightarrow \boxed{\begin{array}{l} (2, 4) \\ (2, -4) \end{array}}$$

For $x = -2$

$$(-2)^2 - y^2 = -12$$

$$4 - y^2 = -12$$

$$y^2 = 16$$

$$y = \pm 4$$

$$\Rightarrow \boxed{\begin{array}{l} (-2, 4) \\ (-2, -4) \end{array}}$$

3) Graph the solution set of the following system of inequalities:

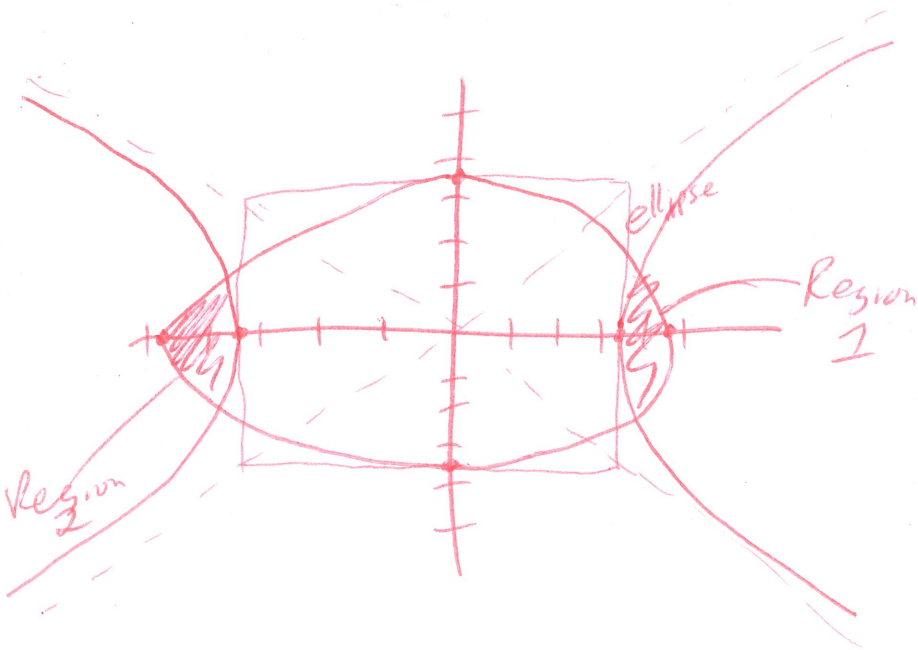
$$\begin{cases} x^2 + 2y^2 \leq 24 \\ -x^2 + y^2 \leq -12 \end{cases}$$

$$\underline{\text{Egn 1}} \Rightarrow \frac{x^2}{24} + \frac{y^2}{12} \leq 1$$

$$\Rightarrow \frac{x^2}{(\sqrt{24})^2} + \frac{y^2}{(\sqrt{12})^2} \leq 1$$

$$\underline{\text{Egn 2}} \Rightarrow \frac{x^2}{(\sqrt{12})^2} - \frac{y^2}{(\sqrt{12})^2} \geq 1$$

$$\begin{aligned} \sqrt{12} &\approx 3.46 \\ \sqrt{24} &\approx 4.89 \end{aligned}$$



Test Egn 1 (0,0)

$$0 + 0 \leq 24 \quad \checkmark$$

Test Egn 2 (0,0)

$$0 + 0 \leq -12 \quad \times$$

Shade outside

4) Solve the following system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{cases} 3x + 7y + 22z = 83 \\ x + 3y + 10z = 37 \\ -2x - 5y - 18z = -66 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 1 & 3 & 10 & 37 \\ -2 & -5 & -18 & -66 \end{array} \right] \xrightarrow{\substack{-3R_2 \\ 3R_3}} \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ -3 & -9 & -30 & -111 \\ -6 & -15 & -54 & -198 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & -2 & -8 & -28 \\ 0 & -1 & -10 & -32 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & 1 & 4 & 14 \\ 0 & -1 & -10 & -32 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & -6 & -18 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 3 & 7 & 0 & 17 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 3 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\boxed{\begin{array}{l} x=1 \\ y=2 \\ z=3 \end{array}}$$

5) Compute $A \cdot B$ for the following matrices:

$$A = \begin{pmatrix} 1 & 4 \\ 8 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 1 & 4 \\ 8 & 3 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1(2) + 4(1) & 5(1) + 6(4) \\ 8(2) + 3(1) & 5(8) + 3(6) \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 29 \\ 19 & 58 \end{pmatrix}$$

6) The matrix A is below. Find its inverse matrix, A^{-1} .

$$A = \begin{pmatrix} 9 & 7 \\ 5 & 3 \end{pmatrix}$$

$$\det A = 27 - 35 = -8$$

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \boxed{-\frac{1}{8} \begin{pmatrix} 3 & -7 \\ -5 & 9 \end{pmatrix}}$$

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST

