

Name: KEY

Score: _____ / 100

Student ID: _____

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	10	Total
✓											27
Score											
Pts. Possible	3	3	3	3	3	3	3	3	3	3	29

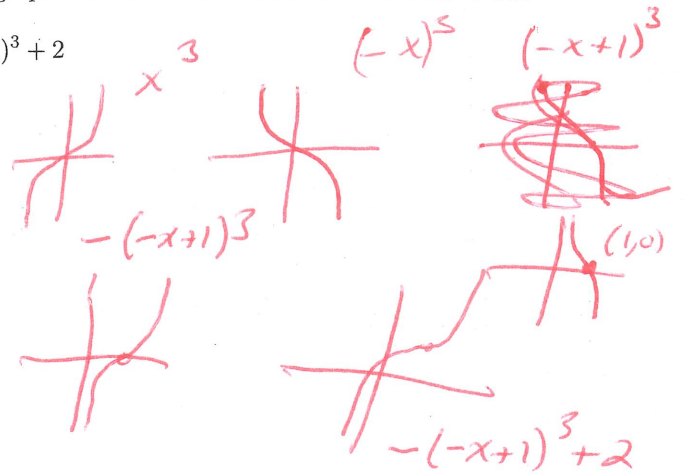
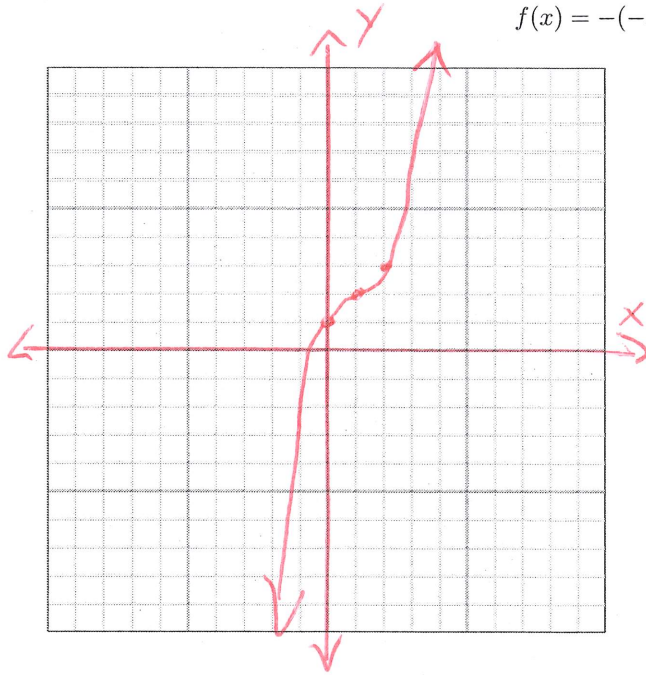
INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 10 question exam.
- Students have 2 hours and 15 minutes to complete the exam.
- The test will be out of **27 points**. The highest possible score will be **29 points**. You must complete 9 problems for credit (3 points each, 27 points total). If you wish, you can attempt a 10th problem for extra credit. That question will be out of 2 points, for a maximum of 29 possible points.
- In the above table, the row with the ✓ should be marked for the 9 questions you want graded. Mark **EC** for the extra credit problem.
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The back of the test can be used for scratch work.

GOOD LUCK!

1) Use graph transformations to sketch the following graph. USE THE GRAPH PAPER FOR YOUR FINAL ANSWER.

$$f(x) = -(-x+1)^3 + 2$$



2) Use polynomial or synthetic division to divide the polynomials:

$$\frac{x^4 - 3x^3 + 3x^2 - 3x + 2}{x - 2}$$

$$\begin{array}{r} 2 \overline{) 1 - 3 \ 3 - 3 \ 2} \\ \underline{ 2 - 2 \ 2 - 2} \\ 1 - 1 \ 1 - 1 \ 0 \end{array}$$

$$\boxed{x^3 - x^2 + x - 1}$$

3) Put the following quadratic function in vertex form. State the vertex and axis of symmetry.

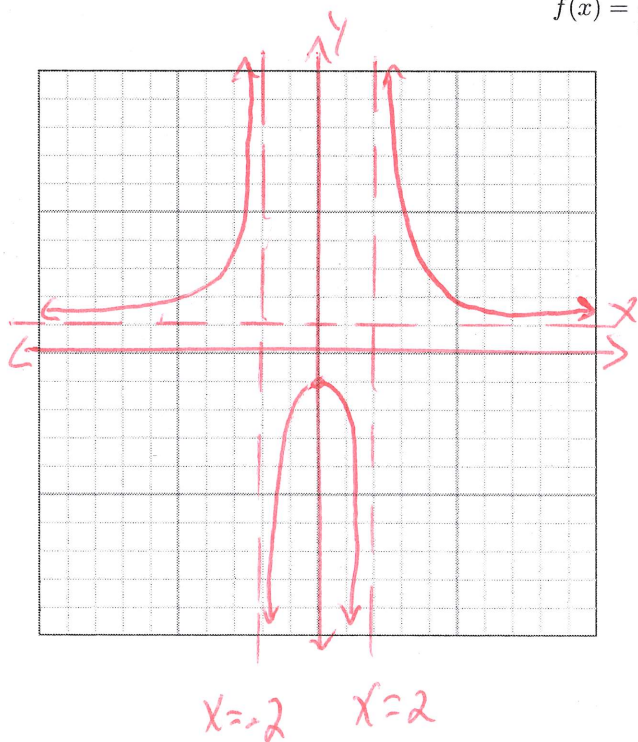
$$f(x) = 2x^2 - 20x + 48$$

$$\begin{aligned} f(x) &= 2x^2 - 20x + 48 \\ &= 2(x^2 - 10x) + 48 \\ &= 2(x^2 - 10x + 25) + 48 - 50 \\ &= \boxed{2(x-5)^2 - 2} \end{aligned}$$

$$\Rightarrow \begin{cases} \text{vertex} = (5, -2) \\ \text{axis of symmetry} \Rightarrow x = 5 \end{cases}$$

4) Identify the asymptotes and zeros of the function. Sketch the graph. **USE THE GRAPH PAPER FOR YOUR FINAL ANSWER.**

$$f(x) = \frac{x^2 + 4}{x^2 - 4}$$



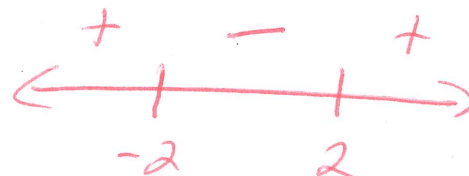
Zeros: None

VA's: $x = \pm 2$

HA's: $y = 1$

Xint: $(0, -1)$

Sign chart



5) Solve the following rational inequality, and put the answer in **interval notation**:

$$\frac{5}{2-x} \leq \frac{3}{3-x}$$

$$\frac{5}{2-x} - \frac{3}{3-x} \leq 0$$

$$\frac{5(3-x) - 3(2-x)}{(2-x)(3-x)} \leq 0$$

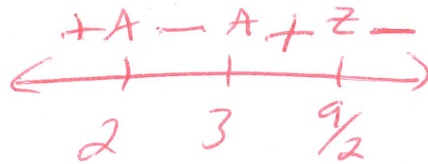
$$\frac{15 - 5x - 6 + 3x}{(2-x)(3-x)} \leq 0$$

$$\frac{-2x + 9}{(2-x)(3-x)} \leq 0$$

Zeros: $-2x + 9 = 0$
 $x = \frac{9}{2}$

VA's: $x = 2, 3$

Sign chart



$$(2, 3) \cup \left[\frac{9}{2}, \infty\right)$$

6) Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ for the function $f(x) = \sqrt{x+2}$, and reduce completely.

$$f(x) = \sqrt{x+2}$$

$$f(x+h) = \sqrt{x+h+2}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2} + \sqrt{x+2}}{\sqrt{x+h+2} + \sqrt{x+2}}$$

$$= \frac{(x+h+2) - (x+2)}{h(\sqrt{x+h+2} + \sqrt{x+2})} = \frac{\cancel{x+h+2} - \cancel{x-2}}{h(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$= \frac{1}{\sqrt{x+h+2} + \sqrt{x+2}}$$

7) Write the inverse function, $f^{-1}(x)$, for $f(x) = 2x + 9$, and check that your result is the inverse.

$$\begin{aligned} y &= 2x + 9 \\ x &= 2y + 9 \\ x - 9 &= 2y \\ y &= \frac{x-9}{2} \end{aligned}$$

$$\boxed{f^{-1}(x) = \frac{x-9}{2}}$$

$$\begin{aligned} f(f^{-1}(x)) &= 2\left(\frac{x-9}{2}\right) + 9 = x - 9 + 9 = x \\ f^{-1}(f(x)) &= \frac{2x+9-9}{2} = \frac{2x}{2} = x \quad \checkmark \end{aligned}$$

Shows f and f^{-1} are inverses

8) Solve the following equation for x : $2e^{2x} + 5e^x - 12 = 0$

$$2e^{2x} + 5e^x - 12 = 0$$

$$\begin{aligned} z &= e^x \\ z^2 &= e^{2x} \end{aligned}$$

$$2z^2 + 5z - 12 = 0$$

$$\begin{aligned} \text{product} &= -24 \\ \text{sum} &= 5 \end{aligned} \quad \text{factors: } 8, -3$$

$$2z^2 + 8z - 3z - 12 = 0$$

$$2z(z+4) - 3(z+4) = 0$$

$$(2z-3)(z+4) = 0$$

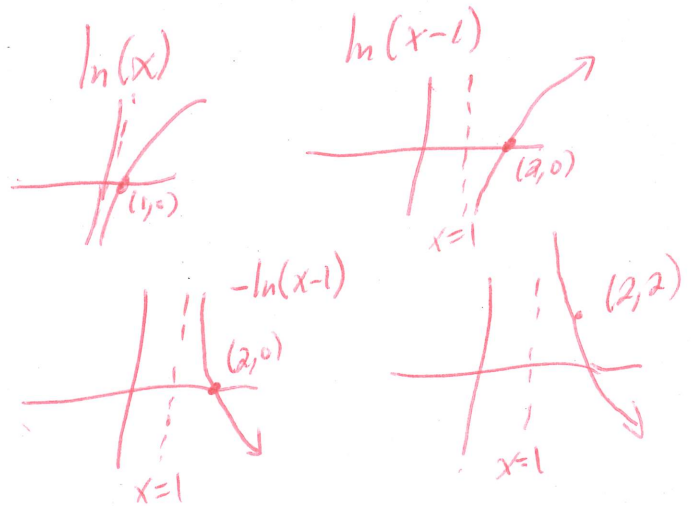
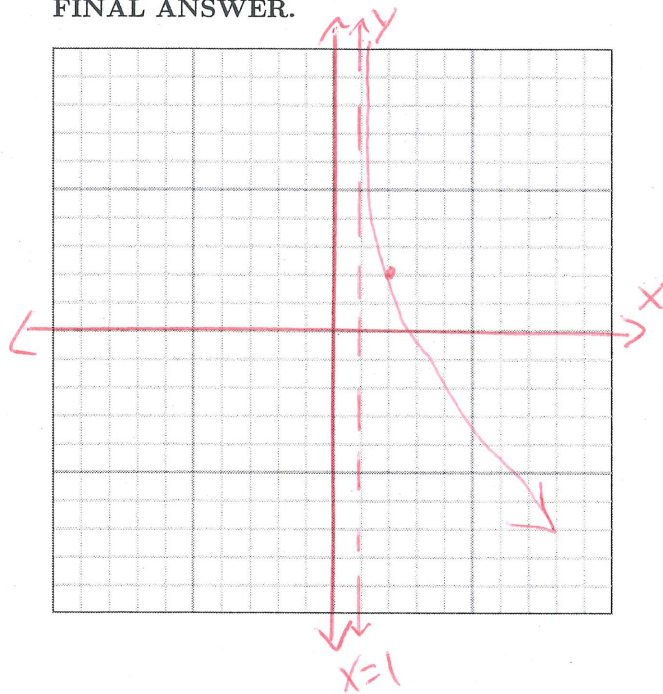
$$z = \frac{3}{2} \quad z = -4$$

$$e^x = \frac{3}{2} \quad e^x = -4$$

$$\boxed{x = \ln\left(\frac{3}{2}\right)}$$

$$x = \cancel{\ln(-4)}$$

9) Graph the following function: $f(x) = -\ln(x-1) + 2$. USE THE GRAPH PAPER FOR YOUR FINAL ANSWER.



10) A bacteria culture is being created and the population is governed by the equation $P(t) = P_0 e^{kt}$, where P_0 and k are constants.

- If the initial population is 20 bacteria, and after 5 minutes there are 100 bacteria, determine k in the $P(t)$ function (Advice: You can leave the \ln in the final answer).
- Find $P(10)$. What does this quantity mean?
- Find the time, t in minutes, to reach 200 bacteria (Advice: You can leave the \ln in the final answer).

$$a) P_0 = 20, t = 5, P(5) = 100$$

$$100 = 20 e^{5k} \Rightarrow 5 = e^{5k} \Rightarrow \ln(5) = 5k$$

$$\boxed{k = \frac{1}{5} \ln(5)}$$

$$b) P(t) = 20 e^{\frac{1}{5} \ln(5) t}$$

$$P(10) = 20 e^{\frac{10}{5} \ln(5)} = 20 e^{2 \ln(5)} = 20 \cdot e^{\ln(25)} = 20 \cdot 25 = \boxed{500}$$

$$c) 200 = 20 e^{\frac{1}{5} \ln(5) t}$$

$$10 = e^{\frac{1}{5} \ln(5) t}$$

$$\ln(10) = \frac{1}{5} \ln(5) t \Rightarrow \boxed{t = 5 \frac{\ln(10)}{\ln(5)}}$$

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST