MATH 25 MIDTERM 2 September 15, 2016

Name: KEY

Score: _____ / 100

Student ID: _____

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	10	Total
✓											27
Score											
Pts. Possible	3	3	3	3	3	3	3	3	3	3	29

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 10 question exam.
- Students have 2 hours and 15 minutes to complete the exam.
- The test will be out of **27 points**. The highest possible score will be **29 points**. You must complete 9 problems for credit (3 points each, 27 points total). If you wish, you can attempt a 10^{th} problem for extra credit. That question will be out of 2 points, for a maximum of 29 possible points.
- In the above table, the row with the ✓ should be marked for the 9 questions you want graded. Mark **EC** for the extra credit problem.
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The back of the test can be used for scratch work.

GOOD LUCK!

1) Solve the following system of linear equations:

$$\begin{cases} 2x + 3y = 5 \\ -5x - 2y = 4 \end{cases}$$

Multiply ① by 5
Multiply ② by 2

$$\Rightarrow \quad \begin{aligned} 10x + 15y &= 25 \\ -10x - 4y &= 8 \\ \hline 11y &= 33 \\ y &= 3 \end{aligned}$$

$2x + 3y = 5 \Rightarrow 2x + 3(3) = 5$

$2x + 9 = 5$

$2x = -4$

$x = -2$

$y = 3$
$x = -2$

2) Solve the following system of linear equations:

$$\begin{cases} -3x + 2y - 5z = -14 \\ 2x - 3y + 4z = 10 \\ x + y + z = 4 \end{cases}$$

Multiply ③ by -2 add to ②
Multiply ③ by 3 add to ①

$2x - 3y + 4z = 10$
$-2x - 2y - 2z = -8$
$-5y + 2z = 2$

$-3x + 2y - 5z = -14$
$3x + 3y + 3z = 12$
$5y - 2z = -2$ ⋆

$$\Rightarrow \begin{cases} -5y + 2z = 2 \\ 5y - 2z = -2 \end{cases}$$

$0 = 0 \Rightarrow$ dependant system

Free variable, let $z = z$

$\Rightarrow$ From ⋆ we have

$5y - 2z = -2$
$5y = 2z - 2$
$\Rightarrow y = \frac{2}{5}(z-1)$

From ③, $x = 4 - y - z$
$x = 4 - \frac{2}{5}(z-1) - z$

$$\Rightarrow \boxed{\begin{aligned} x &= 4 - \tfrac{2}{5}(z-1) - z \\ y &= \tfrac{2}{5}(z-1) \\ z &= z \end{aligned}}$$

or $\boxed{\{(x,y,z) \mid z \text{ free}\}$ w.r.t $\{(4 - \frac{2}{5}(z-1) - z, \frac{2}{5}(z-1), z) \mid z \in \mathbb{R}\}}$

Other solutions possible.

3) Solve the following system of nonlinear equations (*Hint: The graph may be helpful.*):

$$\begin{cases} x^2 + 2y^2 = 9 \\ x^2 - 2y^2 = -7 \end{cases}$$

→ ellipse center (0,0)

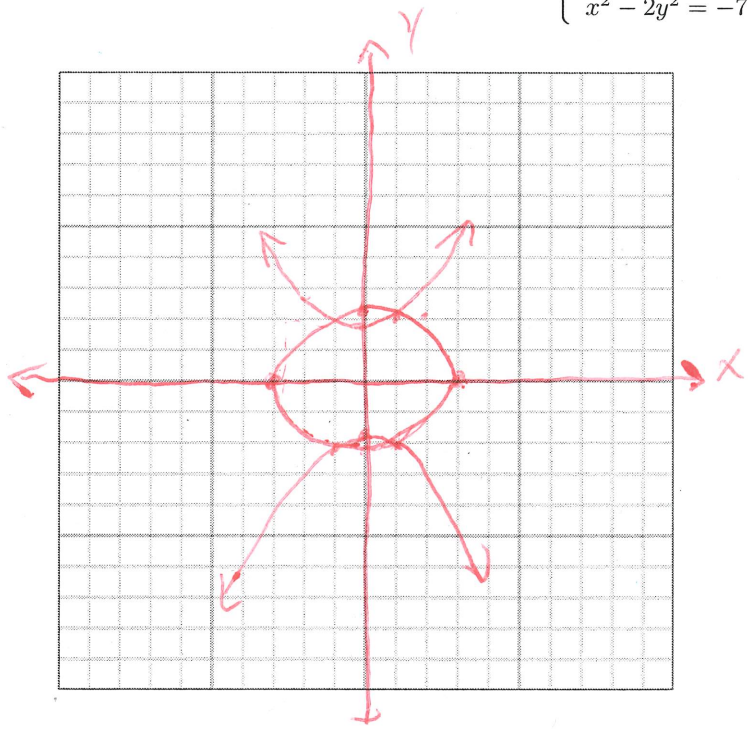
$a = 3$

$b = \sqrt{\frac{9}{2}} \approx 2.1$

hyperbola

$a = \sqrt{7} \approx 2.6$

$b = \sqrt{\frac{7}{2}} \approx 1.8$



$x^2 + 2y^2 = 9$

$\Rightarrow x^2 = 9 - 2y^2$

$\Rightarrow$ From (2) $x^2 - 2y^2 = -7$

$9 - 2y^2 - 2y^2 = -7$

$9 - 4y^2 = -7$

$16 = 4y^2$

$4 = y^2$

$y = \pm 2$

For $y = 2$

$x^2 + 2(2)^2 = 9$

$x^2 + 8 = 9$

$x^2 = 1$

$x = \pm 1$

$\Rightarrow$ $\boxed{(1, 2)}$ $\boxed{(-1, 2)}$

For $y = -2$

$x^2 + 2(-2)^2 = 9$

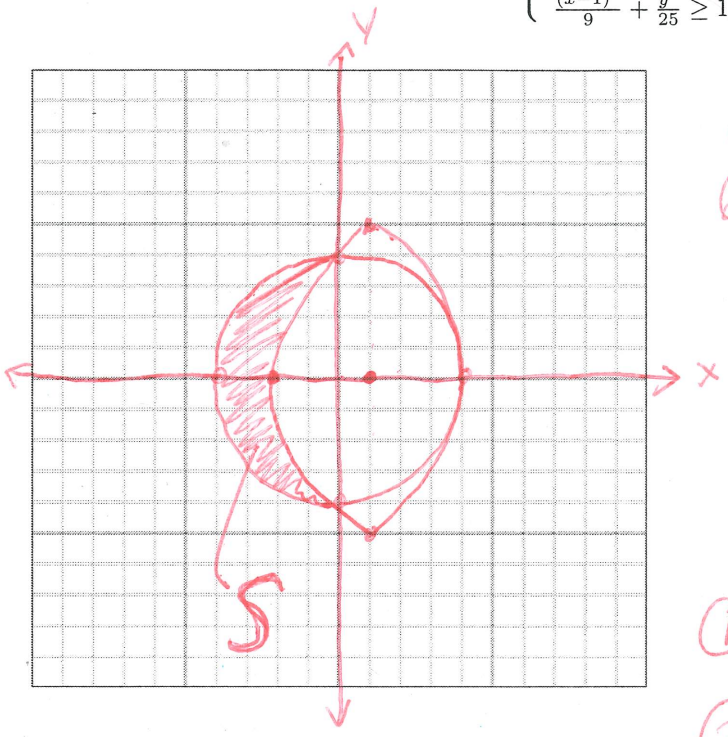
$x^2 + 8 = 9$

$x^2 = \pm 1$

$\Rightarrow$ $\boxed{(1, -2)}$ $\boxed{(-1, -2)}$

4) Graph the solution set of the following system of inequalities:

$$\begin{cases} x^2 + y^2 \leq 16 \\ \frac{(x-1)^2}{9} + \frac{y^2}{25} \geq 1 \end{cases}$$



① circle, r=4, center (0,0)

② ellipse center (1,0)

a=3

b=5

Test origin (0,0) for both

① $0+0^2 \leq 16$ ✓ Shade inside circle

② $\frac{1}{9} \geq 1$ X Shade outside ellipse

5) An algebra test has computational and graphing problems. Computation questions are out of 6 points and graphing questions are out of 10 points. Suppose you can answer computational problems in 2 minutes and graphing questions in 4 minutes. There is at most 40 minutes for the test. You cannot answer more than 12 questions. Assuming you answer all the attempted problems correctly, how many of each type should you answer to get the highest score?

x = computational

y = graphing

$f(x,y) = 6x + 10y$

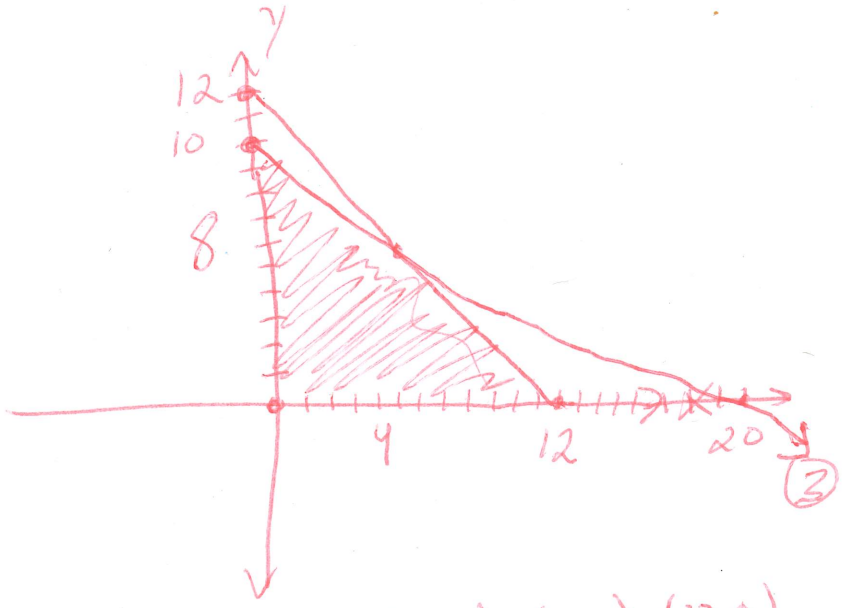
Inequalities

$x \geq 0$

$y \geq 0$

$2x + 4y \leq 40$

$x + y \leq 12$



Vertices: $(0,10), (4,8), (0,0), (12,0)$

(3) $\Rightarrow y = 10 - \frac{1}{2}x$

(4) $\Rightarrow y = 12 - x$

(3) = (4)

$10 - \frac{1}{2}x = 12 - x$

$\frac{1}{2}x = 2$

$x = 4$

$\Rightarrow y = 8$

$(0,0)$ $f(0,0) = 0$

$(4,8)$ $f(4,8) = 6(4) + 8(10) = 104$

$(0,10)$ $f(0,10) = 6(0) + 8(10) = 80$

$(12,0)$ $f(12,0) = 6(12) + 80(0) = 72$

4 computational

8 graphs

6) Solve the following system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{cases} 3x + 7y + 22z &= 83 \\ x + 3y + 10z &= 37 \\ -2x - 5y - 18z &= -66 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 1 & 3 & 10 & 37 \\ -2 & -5 & -18 & -66 \end{array}\right] \xrightarrow[3R_3]{-3R_2} \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ -3 & -9 & -30 & -111 \\ -6 & -15 & -54 & -198 \end{array}\right]$$

$$\xrightarrow[R_1+R_2]{2R_1+R_3} \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & -2 & -8 & -28 \\ 0 & -1 & -10 & -32 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & 1 & 4 & 14 \\ 0 & -1 & -10 & -32 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & -6 & -18 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 3 & 7 & 0 & 17 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 3 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\Longrightarrow \boxed{\begin{array}{l} x = 1 \\ y = 2 \\ z = 3 \end{array}}$$

7) Solve the following system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{cases} x + 3y - 2z &= 5 \\ 3x + 5y + 6z &= 7 \end{cases}$$

$$\begin{bmatrix} 1 & 3 & -2 & 5 \\ 3 & 5 & 6 & 7 \end{bmatrix}$$

$-3R_1+R_2 \Rightarrow$ $\begin{bmatrix} 1 & 3 & -2 & 5 \\ 0 & -4 & 12 & -8 \end{bmatrix}$

$\frac{1}{4}R_2 \Rightarrow$ $\begin{bmatrix} 1 & 3 & -2 & 5 \\ 0 & -1 & 3 & -2 \end{bmatrix}$

$3R_2+R_1 \Rightarrow$ $\begin{bmatrix} 1 & 0 & 7 & -1 \\ 0 & -1 & 3 & -2 \end{bmatrix}$

$-R_2 \Rightarrow$ $\begin{bmatrix} 1 & 0 & 7 & -1 \\ 0 & 1 & -3 & 2 \end{bmatrix} \Rightarrow \begin{aligned} x + 7z &= -1 \\ y - 3z &= 2 \end{aligned}$

z free $\Rightarrow$ $\begin{aligned} z &= z \\ x &= -7z - 1 \\ y &= 3z + 2 \end{aligned}$

or

x free $\Rightarrow$ $\begin{aligned} x &= x \\ y &= \frac{-3}{7}x + \frac{11}{7} \\ z &= -\frac{1}{7}x - \frac{1}{7} \end{aligned}$

8) Compute $B \cdot A$ for the following matrices:

$$A = \begin{pmatrix} 1 & 4 & 2 \\ 1 & 3 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{pmatrix}$$

3×2 $\quad 2\times3$ $\quad 3\times3$

$$\begin{pmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 4 & 2 \\ 1 & 3 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 13 & 2 \\ 3 & 11 & 4 \\ 3 & 10 & 2 \end{pmatrix}$$ Correct

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Incorrect! $A \cdot B = (2\times3)\cdot(3\times2) = 2\times2$

$$\begin{pmatrix} 1 & 4 & 2 \\ 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 11 & 11 \\ 7 & 6 \end{pmatrix}$$ Not correct, not commutative

9) The matrix A is below. Find its determinant, $\det(A)$.

$$A = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$$

Pivot around column (1)

$$\det(A) = 7\begin{vmatrix} 3 & -1 \\ 4 & -2 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix} + (-3)\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix}$$

$$= 7(-6-(-4)) + 0 - 3(-2-3)$$

$$= 7(-2) - 3(-5)$$

$$= -14 + 15$$

$$\det(A) = 1$$

10) Solve the following system of equations using the inverse matrix, A^{-1}.

$$\begin{cases} x+y & =1 \\ x+y+z & =2 \\ y+z & =3 \end{cases}$$

$$A=\begin{bmatrix}1&1&0\\1&1&1\\0&1&1\end{bmatrix} \Rightarrow A^{-1} \Rightarrow \left[\begin{array}{ccc|ccc}1&1&0&1&0&0\\1&1&1&0&1&0\\0&1&1&0&0&1\end{array}\right] \xrightarrow{-R_1+R_2}$$

$$A^{-1} \Rightarrow \left[\begin{array}{ccc|ccc}1&1&0&1&0&0\\0&0&1&-1&1&0\\0&1&1&0&0&1\end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc}1&1&0&1&0&0\\0&1&1&0&0&1\\0&0&1&-1&1&0\end{array}\right] \xrightarrow{-R_3+R_2}$$

$$\left[\begin{array}{ccc|ccc}1&1&0&1&0&0\\0&1&0&1&-1&1\\0&0&1&-1&1&0\end{array}\right] \xrightarrow{-R_2+R_1} \left[\begin{array}{ccc|ccc}1&0&0&0&1&-1\\0&1&0&1&-1&1\\0&0&1&-1&1&0\end{array}\right]$$

$$\Rightarrow A^{-1}=\begin{bmatrix}0&1&-1\\1&-1&1\\-1&1&0\end{bmatrix}$$

$$A\vec{x}=b \Rightarrow (A^{-1}A)\vec{x}=A^{-1}b \qquad b=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\vec{x}=A^{-1}b$$

$$\vec{x}=\begin{bmatrix}0&1&-1\\1&-1&1\\-1&1&0\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\Rightarrow \boxed{\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-1\\2\\1\end{bmatrix}}$$

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST