MATH 25 PRACTICE FINAL October 5, 2016

Name: KEY Score: ________ / 100

Student ID: ______________________________

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	10	Total
✓											27
Score											
Pts. Possible	3	3	3	3	3	3	3	3	3	3	29

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 10 question exam.
- Students have 2 hours and 15 minutes to complete the exam.
- The test will be out of **27 points**. The highest possible score will be **29 points**. You must complete 9 problems for credit (3 points each, 27 points total). If you wish, you can attempt a 10^{th} problem for extra credit. That question will be out of 2 points, for a maximum of 29 possible points.
- In the above table, the row with the ✓ should be marked for the 9 questions you want graded. Mark **EC** for the extra credit problem.
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The back of the test can be used for scratch work.

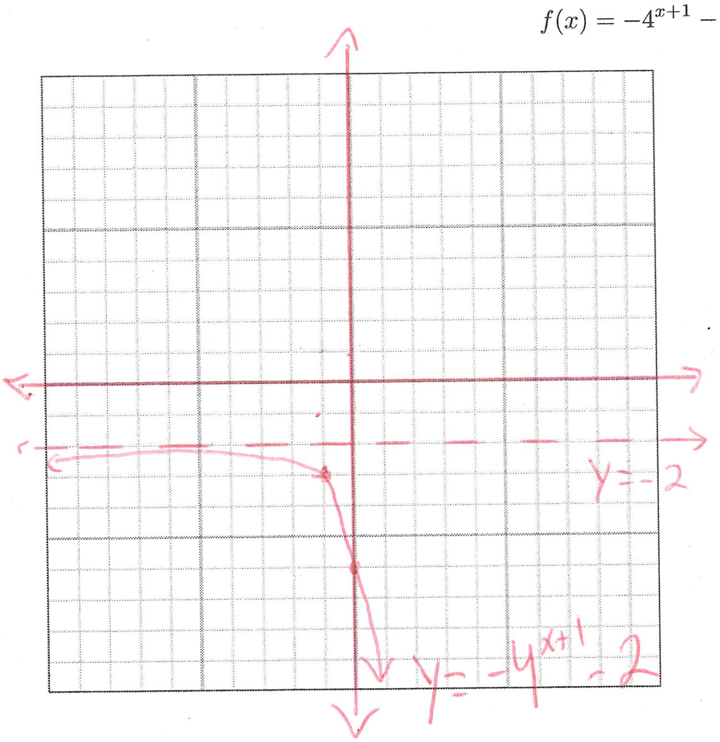
GOOD LUCK!

FORMULAS:

Name	Formula
Foci for ellipse	$c^2 = a^2 - b^2, a > b$
Foci for hyperbola	$c^2 = a^2 + b^2$
n^{th} term of Arithmetic Series	$a_n = a_1 + (n-1)d$
Sum of Arithmetic Series	$S_n = \frac{n}{2}(a_1 + a_n)$
Finite Geometric Series	$\sum_{j=1}^{n} ar^{j-1} = \frac{a(1-r^n)}{1-r}$
Infinite Geometric Series	$\sum_{j=1}^{\infty} ar^{j-1} = \frac{a}{1-r}$
Binomial coefficients	$\binom{n}{r} = \frac{n!}{r! \cdot (n-r)!}$
Binomial Theorem	$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$

1) Graph the following function. Clearly label all asymptotes and the x and y-intercepts.

$$f(x) = -4^{x+1} - 2$$



Base graph $y = 4^x$

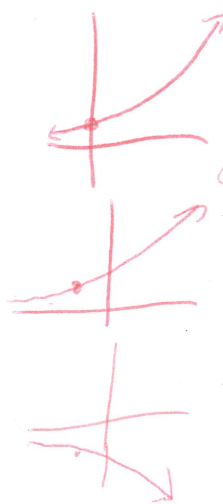
$\downarrow$

$y = 4^{x+1}$ horiz trans 1 unit

$\downarrow$

$y = -4^{x+1}$ reflection over x-axis

$y = -4^{x+1} - 2$ down 2



$0 = -4^{x+1} - 2$

$4^{x+1} = -2 \Rightarrow$ no x ints

$y = -4^{0+1} - 2 = -4 - 2$

$= -6$

$y_{int} = (0, -6)$

2) (a) Solve the following logarithmic equation for $\cancel{x}$ y:

$$\log_3(y) + \log_3(y+6) = 3$$

(b) Solve following exponential equation for x:

$$5^{3x-8} = 25^{2x}$$

(a) $\log_3(y) + \log_3(y+6) = 3$

$\log_3(y(y+6)) = 3$

$\log_3(y^2+6y) = 3$

$\cancel{3^{\log_3}}(y^2+6y) = 3^3$

$y^2 + 6y = 27$

$y^2 + 6y - 27 = 0$

$(y+9)(y-3) = 0 \Rightarrow$ $\cancel{y = -9}$ $\boxed{y = 3}$

(b) $5^{3x-8} = 25^{2x}$

$5^{3x-8} = (5^2)^{2x}$

$5^{3x-8} = 5^{4x}$

$3x - 8 = 4x$

$\boxed{-8 = x}$

3) Solve the following system of equations.

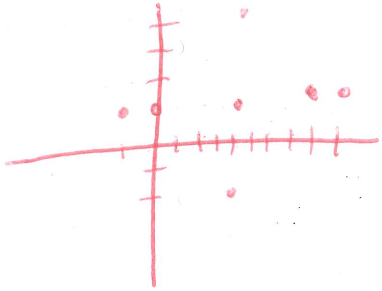
$$\begin{cases} -2x + 5y - 4z = -4 \\ x - 2y + z = 3 \\ x - 5y + 9z = -5 \end{cases}$$

$$\left[\begin{array}{ccc|c} -2 & 5 & -4 & -4 \\ 1 & -2 & 1 & 3 \\ 1 & -5 & 9 & -5 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ -2 & 5 & -4 & -4 \\ 1 & -5 & 9 & -5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & -3 & 8 & -8 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 7 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 2 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 7 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right] \Rightarrow \boxed{\begin{array}{l} x = 4 \\ y = 0 \\ z = -1 \end{array}}$$

4) Write an equation for the ellipse having foci at (0, 1) and (8,1) , a x-vertices at (-1, 1), and (9, 1).



$\Rightarrow$ center is midpoint of vertices or foci

$\Rightarrow$ center $= (4,1)$

$a = 5$ (dist from center to vertex)

$c = 4$ (dist from center to foci)

$c^2 = a^2 - b^2$

$4^2 = 5^2 - b^2 \Rightarrow b^2 = 5^2 - 4^2$

$b^2 = 25 - 16$

$b^2 = 9$

$b = \pm 3$

$$\Rightarrow \boxed{\frac{(x-4)^2}{25} - \frac{(y-1)^2}{9} = 1}$$

5) Write the following equation for the ellipse in standard form. Then identify the values for h, k, a, b.

$$3x^2 + 2y^2 - 30x - 4y + 59 = 0$$

$$(3x^2 - 30x) + (2y^2 - 4y) = -59$$

$$3(x^2 - 10x) + 2(y^2 - 2y) = -59$$

$$3(x^2 - 10x + 25) + 2(y^2 - 2y + 1) = -59 + 75 + 2$$

$$3(x-5)^2 + 2(y-1)^2 = 18$$

$$\frac{(x-5)^2}{6} + \frac{(y-1)^2}{9} = 1$$

$$\boxed{\begin{array}{l} (h,k) = (5,1) \\ a = \sqrt{6} \\ b = 3 \end{array}}$$

6) (a) Find the general term in the sequence:

$$\{a_n\} = \frac{2}{2}, -\frac{5}{4}, \frac{8}{8}, -\frac{11}{16}, \frac{14}{32}, -\frac{17}{64}, \frac{20}{128}, \ldots$$

(b) Find the following sum using the general term found in part (a):

$$\sum_{n=1}^{4} \{a_n\}$$

(a) Alternating: $+, -, +, -, + \Rightarrow (-1)^{n+1}$

Numerator: $2, 5, 8, 11, 14, \ldots$

n	$\cancel{2}1$	$\cancel{3}2$	3	4	5
?	2	5	8	11	14

$\Rightarrow 3n-1$

Denominator: $2, 4, 8, 16, 32 \Rightarrow 2^n$

$$\Rightarrow \boxed{a_n = \frac{(-1)^{n+1}(3n-1)}{2^n}}$$

(b)
$$\sum_{n=1}^{4} \{a_n\} = 1 - \frac{5}{4} + 1 - \frac{11}{16}$$

$$= 2 - \frac{20}{16} - \frac{11}{16}$$

$$= \frac{32}{16} - \frac{20}{16} - \frac{11}{16} = \boxed{\frac{1}{16}}$$

7) Consider the sequence $\{a_j\} = \{1, 6, 11, 16, 21, ...\}$.

a) Identify the type of sequence $\{a_j\}$.
b) What is the value of the term a_{100}?
c) Find the sum of the first 50 terms.

(a) $a_2 - a_1 = 6 - 1 = 5$
$a_3 - a_2 = 11 - 6 = 5$
$a_4 - a_3 = 16 - 11 = 5$
$a_5 - a_4 = 21 - 16 = 5$

$\Rightarrow$ $\boxed{d = 5 \text{ arithmetic}}$

(b) $a_n = a_1 + (n-1)d$

$a_n = 1 + 5(n-1) \Rightarrow a_{100} = 1 + 5(100-1)$
$= 1 + 5(99)$
$\Rightarrow \boxed{a_{100} = 496}$

(c) $S_n = \frac{n}{2}(a_1 + a_n)$

$a_{50} = 1 + 5(50-1)$
$= 1 + 245$
$= 246$

$S_{50} = \frac{50}{2}(1 + 246)$

$S_{50} = 25(247)$

$\boxed{S_{50} = 6175}$

8) Use induction to prove:

$$2+6+10+\ldots+(4n-2)=2n^2 \quad \text{for positive integers } n \geq 1$$

Proof: $\boxed{n=1}$ $(4n-2)=2n^2$ for $n=1$

$(4(1)-2)=2(1)^2$

$4-2=2(1) \Rightarrow 2=2$ ✓

Assume true for $n=k \Rightarrow$ ~~2+6+10+~~

(☆) $2+6+10+\ldots+(4k-2)=2k^2$

$4(k+1)-2=4k+4-2=4k+2$

Prove for $n=k+1$

WTS: $2+6+10+\ldots+(4k-2)+(4k+2)=2(k+1)^2$

From (☆) $\Rightarrow 2+6+10+\ldots+(4k-2)=2k^2$

Add $4k+2$ to both sides

$\Rightarrow 2+6+10+\ldots+(4k-2)+(4k+2)=2k^2+4k+2$

$=2(k^2+2k+1)$

$=2(k+1)^2$ ✓

So Statements is true for all $n \geq 1$ by induction ■

9) Use the binomial theorem to expand $(2x+3y)^5$

Pascals triangle generates binomial coeffs.

Otherwise use $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$\Rightarrow 1\cdot(2x)^5(3y)^0 + 5\cdot(2x)^4(3y)^1 + 10(2x)^3(3y)^2$

$+ 10(2x)^2(3y)^3 + 5(2x)^1(3y)^4 + 1\cdot(2x)^0(3y)^5$

$$\Rightarrow \boxed{(2x+3y)^5 = 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5}$$

10) Find the 8^{th} term in the expansion of $(2a + b^4)^{10}$.

$\binom{n}{k-1}$ is the binomial coeff of the k^{th} term

$\Rightarrow k=8, n=10 \Rightarrow \binom{n}{k-1} = \binom{10}{8-1} = \binom{10}{7}$

$= \frac{10!}{7!\,3!} = \frac{10\cdot 9\cdot 8\cdot \cancel{7!}}{\cancel{7!}(3\cdot 2\cdot 1)} = \frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}^{4}}{\cancel{3}\cdot \cancel{2}\cdot 1}$

$= 10\cdot 3\cdot 4 = 120$

Let $x = 2a$, $y = b^4$

$\binom{n}{k-1} x^{n-(k-1)} y^{k-1} = \binom{10}{7} x^{10-(7)} y^{8-1}$

$= \binom{10}{7} x^3 y^7$

$= \binom{10}{7} (2a)^3 (b^4)^7$

$= 120(8a^3)(b^{28})$

$= \cancel{120} \boxed{960a^3b^{28}}$

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END OF TEST