

Name: KEY

Score: _____ / 100

Student ID: _____

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

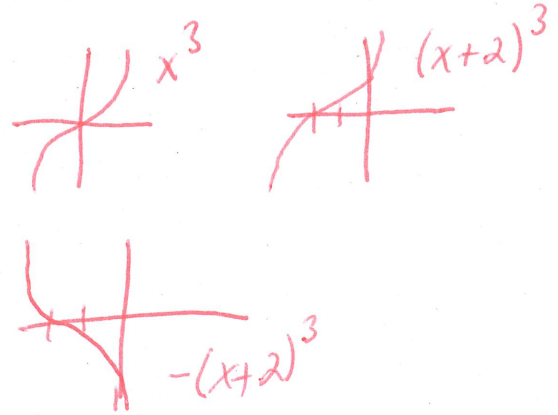
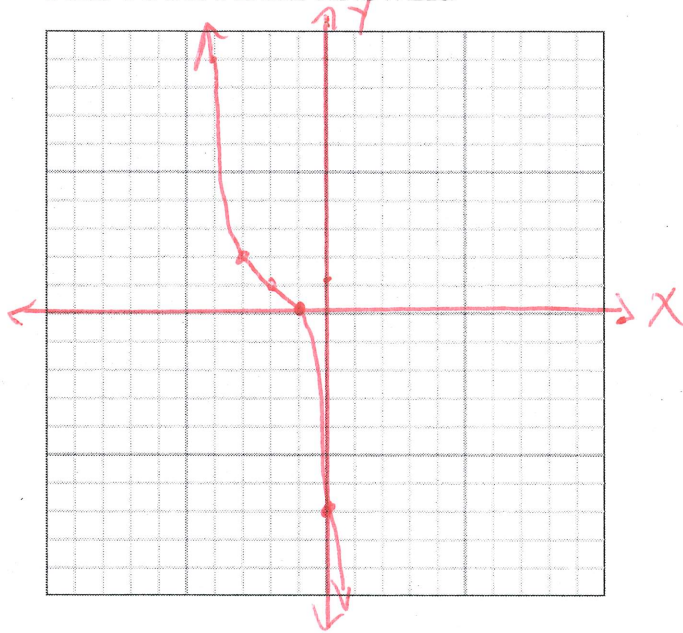
	1	2	3	4	5	6	7	8	9	10	Total
✓											27
Score											
Pts. Possible	3	3	3	3	3	3	3	3	3	3	29

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 10 question exam.
- Students have 2 hours and 15 minutes to complete the exam.
- The test will be out of **27 points**. The highest possible score will be **29 points**. You must complete 9 problems for credit (3 points each, 27 points total). If you wish, you can attempt a 10th problem for extra credit. That question will be out of 2 points, for a maximum of 29 possible points.
- In the above table, the row with the ✓ should be marked for the 9 questions you want graded. Mark **EC** for the extra credit problem.
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The back of the test can be used for scratch work.

GOOD LUCK!

1) Use graph transformations to sketch the graph of $f(x) = -(x+2)^3 + 1$. USE THE GRAPH PAPER FOR YOUR FINAL ANSWER.



2) Use polynomial or synthetic division to divide the polynomials: $(-x^3 + 2x - 21) \div (x - 3)$

$$\begin{array}{r|rrrr} 3 & -1 & 0 & 2 & -21 \\ & \downarrow & -3 & -9 & -21 \\ \hline & -3 & -3 & -7 & -42 \end{array}$$

$$\Rightarrow \boxed{-x^2 - 3x - 7 + \frac{-42}{x-3}}$$

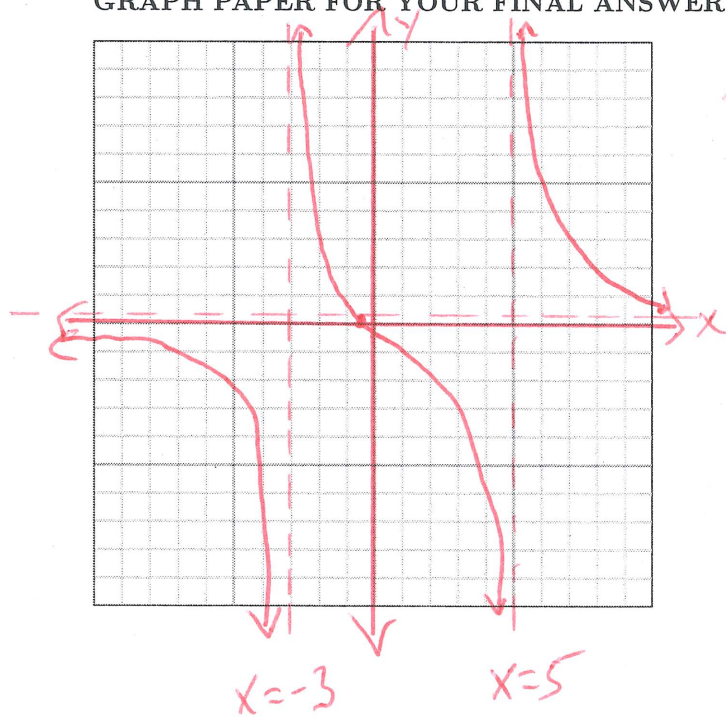
3) Put the following quadratic function in vertex form. State the vertex and axis of symmetry.

$$f(x) = 4x^2 - 24x + 31$$

$$\begin{aligned} f(x) &= 4(x^2 - 6x) + 31 \\ &= 4(x^2 - 6x + 9) + 31 - 36 \\ &= 4(x-3)^2 - 5 \end{aligned}$$

$$\begin{aligned} (h, k) &= 3, -5 \\ \text{a.o.s} &= x = 3 \end{aligned}$$

4) Identify the asymptotes and zeros of the function: $f(x) = \frac{2x+1}{x^2-2x-15}$. Sketch the graph. **USE THE GRAPH PAPER FOR YOUR FINAL ANSWER.**



$$f(x) = \frac{2x+1}{(x-5)(x+3)}$$

Zeros: $x = -\frac{1}{2}$

VA's: $x = 5, x = -3$

HA's: $y = 0$

Sign Chart

-	A	+	Z	-	A	+
-		+		-		+
-	-3	-	$-\frac{1}{2}$	-	5	+

5) Solve the following rational inequality, and put the answer in **interval notation**:

$$\frac{3}{4-x} \leq \frac{6}{1-x}$$

$$\frac{3}{4-x} - \frac{6}{1-x} \leq 0$$

$$\frac{3(1-x) - 6(4-x)}{(4-x)(1-x)} \leq 0$$

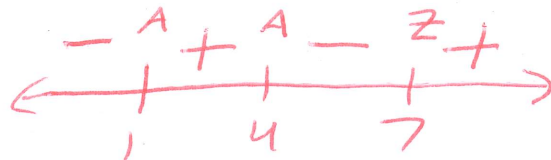
$$\frac{3-3x-24+6x}{(4-x)(1-x)} \leq 0$$

$$\frac{3x-21}{(4-x)(1-x)} \leq 0$$

$$\frac{3(x-7)}{(4-x)(1-x)} \leq 0$$

Zeros: $x=7$

Asymp: $x=1, 4$



$$\Rightarrow \boxed{(-\infty, 1) \cup (4, 7]}$$

6) Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ for the function $f(x) = x^2 - 5x + 3$, and reduce completely.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - 5(x+h) + 3 - (x^2 - 5x + 3)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - 5x - 5h + 3 - \cancel{x^2} + \cancel{5x} - 3}{h}$$

$$= \frac{2xh + h^2 - 5h}{h} = \frac{h(2x + h - 5)}{h}$$

$$= \boxed{2x - 5 + h}$$

7) Write the inverse function, $f^{-1}(x)$, for $f(x) = x^2 + 9$, and check that your result is the inverse.

$$y = x^2 + 9$$

$$x = y^2 + 9$$

$$x - 9 = y^2$$

$$y = \pm \sqrt{x - 9}$$

$$f^{-1}(x) = \sqrt{x - 9}$$

$$\text{Domain of } f(x) = (-\infty, \infty) \Rightarrow \text{Not 1-1}$$

$$f(x) = [0, \infty)$$

$$\text{Range } f(x) = [9, \infty)$$

$$\text{Domain } f^{-1}(x) = [9, \infty)$$

$$\text{Range } f^{-1}(x) = [0, \infty)$$

$$f(f^{-1}(x)) = (\sqrt{x-9})^2 + 9 = x - 9 + 9 = x \quad \checkmark$$

$$f^{-1}(f(x)) = \sqrt{x^2 + 9 - 9} = \sqrt{x^2} = x \quad \checkmark$$

8) Solve the following equation for x : $2e^{2x} + 5e^x - 3 = 0$

$$\text{Let } u = e^x \Rightarrow u^2 = e^{2x}$$

$$\Rightarrow 2e^{2x} + 5e^x - 3 = 0$$

$$\Rightarrow 2u^2 + 5u - 3 = 0$$

$$2u^2 + 6u - u - 3 = 0$$

$$2u(u+3) - (u+3) = 0$$

$$(u+3)(2u-1) = 0$$

$$u = \frac{1}{2}, u = -3$$

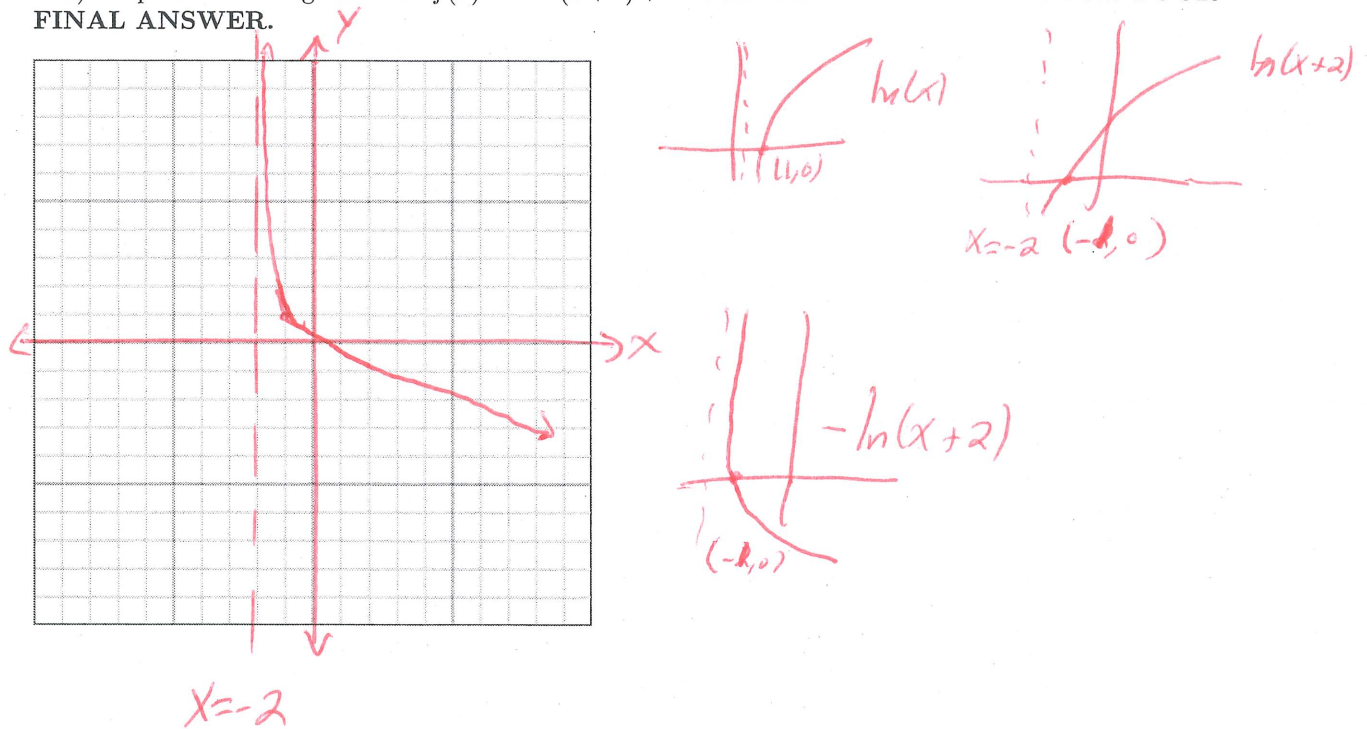
$$e^x = \frac{1}{2} \quad e^x = -3$$

$$\ln e^x = \ln\left(\frac{1}{2}\right)$$

$$x = \ln\left(\frac{1}{2}\right) \text{ or } -\ln(2)$$

Multiply to -6
add to 5 $\Rightarrow 6, -1$

9) Graph the following function: $f(x) = -\ln(x+2) + 1$. USE THE GRAPH PAPER FOR YOUR FINAL ANSWER.



10) A bacteria culture is being created and the population is governed by the equation $P(t) = P_0 e^{kt}$, where P_0 and k are constants.

- If the initial population is 15 bacteria, and after 5 minutes there are 30 bacteria, determine k in the $P(t)$ function.
- Find $P(10)$. What does this quantity mean?
- Find the time, t in minutes, to reach 45 bacteria.

$$a) P_0 = 15; t = 5 \Rightarrow P(5) = 30$$

$$30 = 15 e^{5k} \Rightarrow 2 = e^{5k} \Rightarrow \ln(2) = 5k$$

$$\Rightarrow k = \boxed{\frac{\ln(2)}{5}}$$

$$b) P(10) = 15 e^{\frac{\ln(2)}{5} \cdot 10} = 15 e^{2 \ln(2)} = 15 \cdot 4 = \boxed{60}$$

$$c) 45 = 15 e^{\frac{\ln(2)}{5} t} \Rightarrow 3 = e^{\frac{\ln(2)}{5} t}$$

$$\Rightarrow \ln(3) = \frac{\ln(2)}{5} t$$

$$\Rightarrow \boxed{t = 5 \frac{\ln(3)}{\ln(2)}}$$

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST