MATH 25 PRACTICE MIDTERM 2 September 15, 2016

Name: KEY Score: _____ / 100

Student ID: _____

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	10	Total
✓											27
Score											
Pts. Possible	3	3	3	3	3	3	3	3	3	3	29

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 10 question exam.
- Students have 2 hours and 15 minutes to complete the exam.
- The test will be out of **27 points**. The highest possible score will be **29 points**. You must complete 9 problems for credit (3 points each, 27 points total). If you wish, you can attempt a 10^{th} problem for extra credit. That question will be out of 2 points, for a maximum of 29 possible points.
- In the above table, the row with the ✓ should be marked for the 9 questions you want graded. Mark **EC** for the extra credit problem.
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The back of the test can be used for scratch work.

GOOD LUCK!

1) Solve the following system of linear equations:

$$\begin{cases} 2x - 3y &= -2 \\ 4x + y &= 24 \end{cases}$$

$-2(2x-3y=-2) \Rightarrow -4x+6y=4$

$4x+y=24$

$+$

$7y=28$

$y=4$

$4x+y=24$

$4x+4=24$

$4x=20$

$x=5$

$x=5$
$y=4$

2) Solve the following system of linear equations:

$$\begin{cases} 2x + 7y + 11z = 11 \\ x + 2y + 8z = 14 \\ x + 3y + 6z = 8 \end{cases}$$

$-2R_2, -2R_3$ add to R_1

$2x + 7y + 11z = 11$
$-2x - 4y - 16z = -28$
$3y - 5z = -17$

$2x + 7y + 11z = 11$
$-2x - 6y - 12z = -16$
$y - z = -5$

$\begin{cases} 3y - 5z = -17 \\ y - z = -5 \end{cases}$

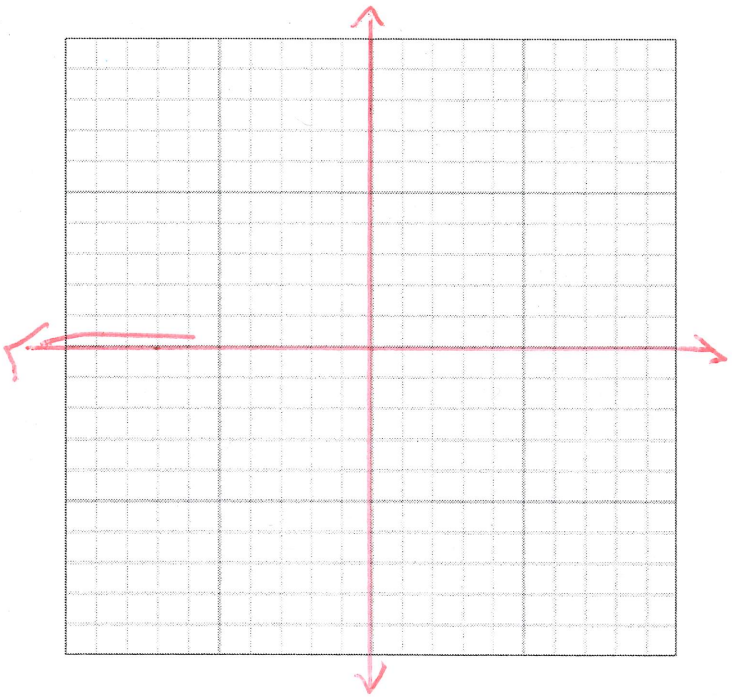
$-3R_2 + R_1$

$3y - 5z = -17$
$-3y + 3z = 15$
$-2z = -2$
$z = 1$

$y - z = -5$
$y = z - 5$
$y = 1 - 5$

3) Solve the following system of nonlinear equations (*Hint: The graph may be helpful.*):

$$\begin{cases} 2x^2 + y^2 = 24 \\ x^2 - y^2 = -12 \end{cases}$$



$2x^2 + y^2 = 24$
$x^2 - y^2 = -12$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

For $x = -2$

$(-2)^2 - y^2 = -12$

$4 - y^2 = -12$

$y^2 = 16$

$y = \pm 4$

For $x = 2$

$(2)^2 - y^2 = -12$

$4 - y^2 = -12$

$y^2 = 16$

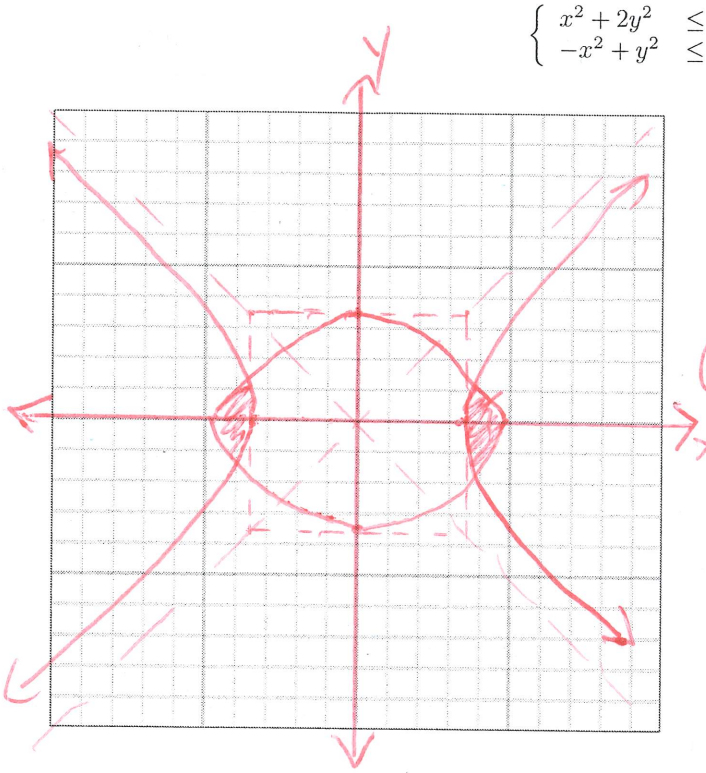
$y = \pm 4$

$(-2, 4)$ $(2, 4)$

$(-2, -4)$ $(2, -4)$

4) Graph the solution set of the following system of inequalities:

$$\begin{cases} x^2 + 2y^2 & \leq 24 \\ -x^2 + y^2 & \leq -12 \end{cases}$$



① $\Rightarrow \frac{x^2}{(\sqrt{24})^2} + \frac{y^2}{(\sqrt{12})^2} \leq 1$

② $\frac{x^2}{(\sqrt{12})^2} - \frac{y^2}{(\sqrt{12})^2} \geq 1$

$\sqrt{12} \approx 3.46$

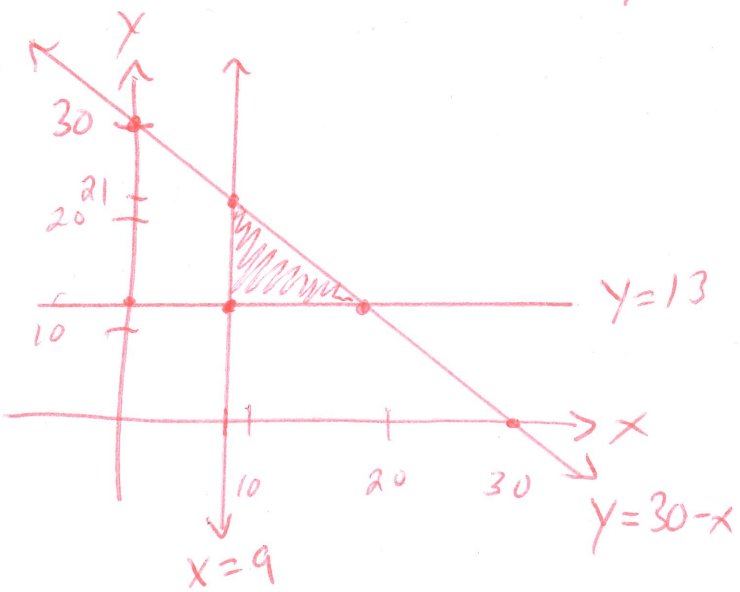
$\sqrt{24} \approx 4.89$

5) A carpentry shop makes tables and and desks. Each week, the shop has to complete at least 9 tables and 13 desks. The shop can make at most 30 tables and desks combined, per week. If the shop sells tables for $ 120 and desks for $ 150, how many of each should be made to maximize weekly income for the shop?

x = tables
y = desks

$x \geq 9$
$y \geq 13$
$x + y \leq 30$

$f(x,y) = 120x + 150y$



Vertices:

(9, 13)
(9, 21)
(17, 13)

$f(9,13) = 120(9) + 150(13) = 3030$

$f(9,21) = 120(9) + 150(21) = 4230$

$f(17,13) = 120(17) + 150(13) = 3990$

$\Rightarrow$ 9 tables, 21 desks

6) Solve the following system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{cases} 3x + 7y + 22z &= 83 \\ x + 3y + 10z &= 37 \\ -2x - 5y - 18z &= -66 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 1 & 3 & 10 & 37 \\ -2 & -5 & -18 & -66 \end{array}\right] \xrightarrow[3R_3]{-3R_2} \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ -3 & -9 & -30 & -111 \\ -6 & -15 & -54 & -198 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & -2 & -8 & -28 \\ 0 & -1 & -10 & -32 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & 1 & 4 & 14 \\ 0 & -1 & -10 & -32 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & -6 & -18 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 3 & 7 & 22 & 83 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 3 & 7 & 0 & 17 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 3 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \Longrightarrow \boxed{\begin{array}{l} x = 1 \\ y = 2 \\ z = 3 \end{array}}$$

7) Solve the following system using Gaussian elimination or Gauss-Jordan elimination.

$$\begin{cases} x - 3y - 17z = -17 \\ -2x + 7y + 38z = 40 \end{cases}$$

$2x - 6y - 34z = -34$

$-2x + 7y + 38z = 40$

$y + 4z = 6$

$$\left[\begin{array}{ccc|c} 1 & -3 & -17 & -17 \\ -2 & 7 & 38 & 40 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -3 & -17 & -17 \\ 0 & 1 & 4 & 6 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -5 & 1 \\ 0 & 1 & 4 & 6 \end{array}\right]$$

$x - 5z = 1$

$y + 4z = 6$

$y = 6 - 4z$

$x = 1 + 5z$

$z = z$

8) Compute $A \cdot B$ for the following matrices:

$$A = \begin{pmatrix} 1 & 4 \\ 8 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 1 & 4 \\ 8 & 3 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1(2)+4(1) & 5(1)+6(4) \\ 8(2)+3(1) & 5(8)+3(6) \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 6 & 29 \\ 19 & 58 \end{pmatrix}}$$

9) The matrix A is below. Find its determinant, $\det(A)$.

$$A = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$$

Pivot around (7)

$$\det(A) = 7\begin{vmatrix} 3 & -1 \\ 4 & -2 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix} + (-3)\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix}$$

$$= 7(-6-(-4)) + 0 - 3(-2-3)$$

$$= 7(-2) - 3(-5)$$

$$= -14 + 15$$

$$\det(A) = 1$$

10) The matrix A is below. Find its inverse matrix, A^{-1}.

$$A = \begin{pmatrix} 9 & 7 \\ 5 & 3 \end{pmatrix}$$

For 2x2, $A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$\det(A) = 9(3) - 7(5) = 27 - 35 = -8$

$\Rightarrow \boxed{A^{-1} = \frac{-1}{8} \begin{pmatrix} 3 & -7 \\ -5 & 9 \end{pmatrix}}$

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST