§1.9 - **Simplifying Algebraic Expressions**

Commutative Property: $a + b = b + a$ and $a \cdot b = b \cdot a$
Associative Property: $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Distributive Property: $a \cdot (b + c) = a \cdot b + a \cdot c$

Example 0.1. *Simplify the following:*
a) $-9 \cdot 3b$
b) $15a(6)$
c) $4b(5a)$
d) $\frac{15}{4} \cdot \frac{12}{5}r$
e) $-6(2y - 2)$
f) $3(-3y^2 - 4)$
g) $\frac{2}{3}(3x + 9y)$
h) $-(6y - 3)$

Combining like terms: Two expressions that look similar can be combined together. For example, $3x+5x$ can be combined to give $8x$. On the other hand, things like $3x + 4y$ cannot be combined because x and y are different variables. Therefore, we leave it as $3x + 4y$.

Example 0.2. *Simplify the following:*
a) $3x + 6y + 5x + 10y - x$
b) $2x + y - 2z - 5x + 6y + 3z$
c) $4x + 6y - 3z^2 + 2z - 6x - 2y + z^2$
d) $5x^2 - 6y^2 - 10x^2 - 4x - 2y - 3 + 10$
e) $4x^2 - z^3 + 10z^2 - 4x + 10 - 3x + 11$

§2.1 - **Solving Equations**

Example 0.3. *Solve the following for x:*
a) $x - 10 = 2$
b) $x + 5 = -2$
c) $3x - 5 = 7$
d) $x + \frac{1}{8} = \frac{7}{4}$
e) $\frac{x}{4} = 4$
f) $\frac{x}{4} = \frac{6}{12}$
g) $-\frac{5x}{4} = \frac{3}{16}$
h) $-\frac{6x}{5} = \frac{10}{3}$

Practice Problems
1) Simplify: $4z^2 + 3x - 4y + 3t^2 + 4x - z^2 - t^2 + 4 + 10 + (3t - 1)^2$
2) Solve for x: $2x + 5 - 3x + 2 = 8$
3) Solve for x: $-\frac{6}{5}(x - 5) = -5$

§2.2 - **More on Solving Equations**

We will need the following definitions about linear equations later, so we will give them now:
A linear equation can be written in standard form as: $ax + by = c$
A linear equation can be written in point-slope form as: $y - y_1 = m(x - x_1)$
A linear equation can be written in slope-intercept form as: $y = mx + b$

Solve:

Example 0.4. *a)* $-12x + 5 = 17$
b) $\frac{3}{4}x - 1 = 5$
c) $-0.2 = -0.8 - y$
d) $4x + 3 = -9$
e) $\frac{1}{6}x + \frac{1}{4}x = 1$
f) $9(x - 1) = 6(x + 2) - x$
g) $2n - \frac{3}{4}n = \frac{1}{2}n + \frac{13}{3}$
h) $3(A + 2) = 2(A - 7)$
i) $4(a - 3) = -2(a - 6) + 6a$ ***
j) $\frac{1}{3}(7 - 7x) = 21$
k) $\frac{3}{4}(d - 8) = \frac{2}{3}(d + 1)$
l) $-\frac{3}{5}(10x + 25) = \frac{4}{7}(14x + 49)$
m) $-\frac{2}{3}(-x + 9) = -\frac{3}{2}(4x - 2)$

§2.3 - **Percents**

The key thing we need to do in the math for percents is to change percentages to decimals. Whenever we are given percents, we move the decimal over in the percentage by two spaces. Some examples are

$$95\% = .95$$
$$2\% = 2.0\% = .02$$
$$.01\% = .0001$$

Now here are some word problems so that we can see how to use percents in some applications:

Example 0.5. *A* 30% *discount on a 1 year membership for a fitness center amounted to a* 90% *savings. Find the cost of the membership before the discount.*

Example 0.6. *A real estate agent earned* $\$14,025$ *for sell a house. If she received a* $5\frac{1}{2}\%$ *commision, how much did the house sell for?*

Example 0.7. *A genealogist determined that worldwide, 180 out of 10 million people had the same last name as his. What percentage is this?*

Example 0.8. *An art gallery agreed to sell an artist's painting for a commsion of* 45%*. What must the selling price of the painting be if the gallery wants to make* $\$13,500$*?*

§2.4 - **Formulas**

Special Formulas to know

$r = c + m$	Retail Price
$p = r - c$	Profit
$I = Prt$	Interest
$d = rt$	Distance
$P = 2l + 2w$	Perimeter of rectangle
$A = \pi r^2$	Area of circle
$V = \pi r^2 h$	Volume of a cylinder

Solve for y or the indicated variable:

Example 0.9. *a)* $3x + 4y = 1$
b) $-2x - 5y = 10$
c) Solve perimeter formula for w
d) $\frac{3}{4}x + \frac{1}{2}y = 6$
e) $\frac{1}{6}x + \frac{1}{4}y = 1$
f) $S = 2\pi(r^2 + rh)$ *Solve for* h *(Challenge: solve for* r*)*
g) $K = \frac{1}{2}mv^2$ *Solve for* v
h) $E = mc^2$ *Solve for* c
i) $T = 2r + 2t$ *Solve for* r

§2.5 - **Problem Solving**

Example 0.10. *A trucking company had its logo embroidered on thr front of baseball caps. It was charged* \$8.90 *per hat plus a one time fee of* \$25*. If the project cost* \$559*, how many hats were embroidered?*

Example 0.11. *A classic car owner is going to sell his 1959 Chevy Impala at auction. He wants to make* $\$46,000$ *after paying an* 8% *commission to the auctioneer What should the selling price of the car be?*

Example 0.12. *The year George Washington was chosen president and the year the Bill of Rights went into effect are consecutive odd integers whose sum is 3580. Find the years.*

Example 0.13. *Police used 400 feet of yellow tape to fence off a rectangular lot for an investigation. They used 50 fewer feet of tape for each width as they did for each length. Find the dimensions of the lot.*

Example 0.14. *If the vertex angle of an isosceles triange is 56 degrees, find the measure of each base angle.*

§2.6 - **More Problem Solving**

Example 0.15. *A college student wants ton invest the* $\$12,000$ *inheritance he received and use the annual interest earned to pay his tuition cost of* \$945*. The highest rate offered by the bank is* 6% *interest. At this rate, he cannot earn the needed* \$945*, so he decides to invest in a riskier, more profitable investment offering* 9% *return. How much should he invest at each rate?*

Example 0.16. *A cargo ship, heading into port, radios the Coast Guard that it is experiencing engine trouble and that its speed has dropped to 3 mph. Immediately the Coasts Guard ship leaves and speeds at 25 mph towards the disabled ship which is 56 miles away. How long will it take for the Coast Guard ship to reach the boat?*

Example 0.17. *While on tour, a country music star travels by bus. Her musical equipment is carried in a truck. How long will it take for her bus traveling at 60 mph to overtake the truck traveling at 45 mph, if the truck had a 1.5 hour head start to her next concert?*

Example 0.18. *A chemistry experiment calls for a* 30% *sulfuric acid solution. If the lab room only has* 50% *and* 20% *solution, how much of each is needed in order to make 12 liters of* 30% *solution?*

Example 0.19. *Colombian coffee costs 9 dollars per pound and French coffee costs 6 dollars per pound. How many pounds of French should be mixed with 50 pounds of Colombian to make a mixture that costs 7 dollars per pound?*

Example 0.20. *A restaurant owner needs to purchase some tables, chairs, and plates for the dining area. She plans on buying 4 chairs and 4 plates for each table. She also plans on buying an additional 20 plates in case of a breakage. If a table costs 100 dollars, chairs cost 50 dollars, and plates cost 5 dollars. How many of each item can she buy for 6500 dollars?*

Example 0.21. *A cyclist leaves his training base for a morning workout riding at 18 mph. Then 1.5 hours later, his support stagg leaves at 45 mph in the same direction. How long will it take for the the support staff to catch up to the cyclist?*

§2.7 - **Solving Inequalities**

A linear inequality in one variable can be written as one of the following:

$$\begin{cases} ax+b > c \\ ax+b \geq c \\ ax+b < c \\ ax+b \leq c \end{cases}$$

Example 0.22. *Is 9 a solution of* $2x+4 \leq 21$*?*

Example 0.23. *Graph on a number line* $x \leq 2$*.*

Example 0.24. *Graph on a number line* $-4 \leq x < 0$*.*

Example 0.25. *Solve* $x+3 > 2$ *and write the solution in interval notation.*

For the following, solve the inequality, write the solution in interval notation, and graph it.

Example 0.26. $-\frac{3}{2}t \geq -12$

Example 0.27. $-5t \geq 55$

Example 0.28. $-5 > 3x+7$

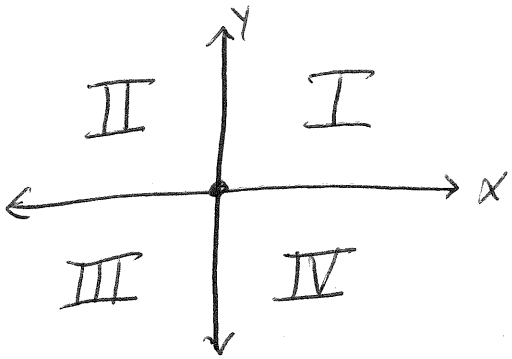
Example 0.29. $8(y+1) \geq 2(y-4)+y$

Example 0.30. $5(b-2) \geq -(b-3)+2b$

Example 0.31. $\frac{3}{4}+\frac{x}{2} > \frac{6}{7}$

Example 0.32. $-4 < 2(x-1) \leq 4$

Section 3.1 - Graphing Using Rectangular Coordinates



Rectangular coordinates
= Cartesian Coordinates

* x, y axes
* origin (0,0)
* Quadrants go counter clockwise
* (x,y) = (x coordinate, y coordinate)

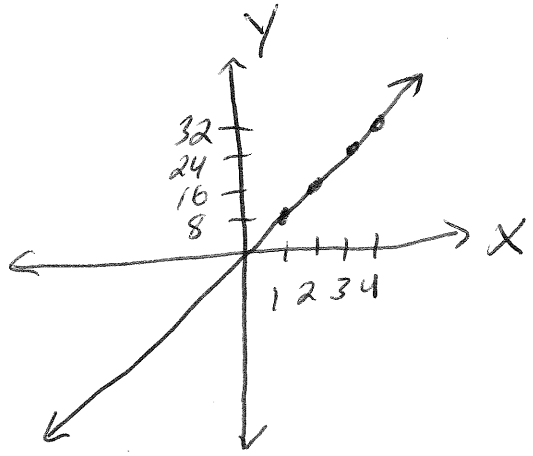
Ex) Plot the points and state quadrant

1) (4,4)
2) $(-1, -\frac{7}{2})$
3) (0, 2.5)
4) (-3,0)
5) (0,0)

Graph Paired Data

Ex)

Time (min)	Water (gal)
0	0
1	8
2	16
3	24
4	32



Ex) See Example 3, p. 190-191 in the text.

Ex) Plot data

t	dist
0	0
2	10
4	20
8	40

What is the distance at $t = 5$?

3.2 - Graphing Linear Equations

Ex) Is $(-1,-2)$ a solution to $y = x-1$?

Ex) Is $(-5,2)$ a solution of $y = 5x+1$?

For the following, complete the solution:

$(-5, \_\_)$ solves $y = -2x+3$

$(-2, \_\_)$ solves $y = 4x-2$

Ex) Consider $3x+2y=5$, Complete the tables:

x	y	(x,y)
7	—	(7,)
—	4	(, 4)

and

x	y	(x,y)
—	-2	(, -2)
5	—	(5,)

For $x-2y=8$

x	y	(x,y)
3	5	
4		

For $8x-4y=-16$

x	y	(x,y)
2	3	
4		

Construct a table of values for the following:

1) $3x+4y=6$ for 5 values

2) $2x-3y=1$ for 4 values

3) $3x-5y=10$ for 3 values

Use the tables from 1,2,3 above to graph the lines.

Standard Form: $Ax + By = C$

Linear functions have exponent 1 for each variable.

Are the following functions linear?

$x^2 + 3$

$x^2 + y^2 = 25$

$y = \frac{1}{x}$

$y = 2x + 5$

Plot $y = -3x$

Plot $2x + 3y = -12$ by solving for y.

Section 3.3 - Intercepts

Ex) $y = 2x - 4$ find x, y ints (graphically)

x-intercept - Where graph crosses x-axis

y-intercept - Where graph crosses y-axis

Graph $y = 3$, $y = x + 1$, $y = 3 - x$

To find intercepts:

y-intercept: Plug in 0 for x and solve for y, $(0, y)$

x-intercept: " " for y and " " x, $(x, 0)$

Ex) Find intercepts of

$x - 3y = 6$

$4x - 2y = 12$

$2x + 5y = -1$

$3x = -5y + 8$

$8x = -4y + 15$

Horizontal Lines

$y = 3$, $y = 4$, $y =$ number

For any x, y stays the same

Graph $y = 3$
$y = -1$

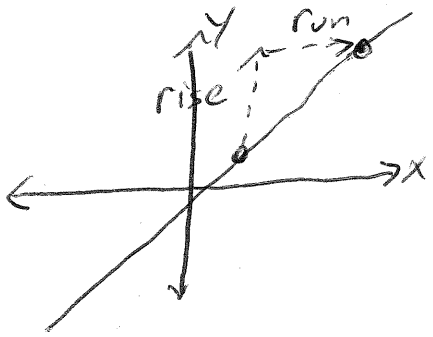
Vertical Lines

$x = 3$, $x = -1$, $x =$ number

For any y, x stays same

Graph $x = 2$
$x = -3$

3.4 - Slope



rise = vertical change
run = horizontal change

$y = mx + b$ (Slope-intercept form)

m = slope
& b = y-int

$$m = \text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

for $x_2 \neq x_1$

Steps

(1) Pick two points on the line

(2) Draw The triangle between points

(3) Find $\frac{\text{rise}}{\text{run}} = m$

rise $\uparrow +$ $\downarrow -$ run $\rightarrow +$ $\leftarrow -$

Alternatively if given (x_1, y_1) and (x_2, y_2)

use $m = \frac{y_2 - y_1}{x_2 - x_1}$

Ex) $(1,2)$ and $(3,8)$ $\quad$ $(-1,-2)$ and $(1,-7)$

$(-2,4)$ and $(5,-6)$ $\quad$ $(3,8)$ and $(4,-3)$

$m > 0 \Rightarrow$ positive slope

$m < 0 \Rightarrow$ negative slope

Slope for horizontal line is $m = 0$

" " vertical line is $m =$ undefined.

Parallel Lines: slopes are equal $(m_1 = m_2)$

Perpendicular Lines: slopes are negative reciprocals $(m_1 \cdot m_2 = -1)$

Ex) Using the two sets of coordinates to see if the lines are perpendicular, parallel or neither.

(A) Line 1 $(-1,8)$ and $(-6,8)$

Line 2 $(3,3)$ and $(3,7)$

(B) $(6,4)$ and $(2,5)$

$(-2,-3)$ and $(2,-4)$

Section 3.5 - Slope Intercept Form

Write the following lines in slope intercept form. State m, and y-int.

1) $8x + y = 9$

2) $x + 4y = 16$

3) $-9x - 3y = 11$

4) $6y = 36$

Ex) Graph the 4 lines above.

Exercise: Graph

$y = 5x - 4$
$y = 2x - 3$

$4x + 3y = 6$
$6x - 2y = 14$

Use Slope intercept form to see if the lines below are parallel, perpendicular or neither

(A) $y = -5x + 6$
$y = 4x + 6$

(B) $x - 5y = -10$
$x - 4y = -8$

Word Problems: See Example on p. 236.

Section 3.6 - Point Slope

Point Slope Formula: $y - y_1 = m(x - x_1)$

Ex) Write equation of the line given

a) $m = -8$, $(-1, 5)$
b) $m = \frac{2}{3}$, $(5, -2)$
c) $m = \frac{-3}{2}$, $(6, -9)$
d) $m = -\frac{1}{4}$, $(-3, -4)$

Ex) Calculate m then use point slope

a) $(-2, 6)$ and $(4, 7)$
b) $(-5, 4)$ and $(8, -6)$
c) $(3, -2)$ and $(4, 1)$
d) $(-6, 3)$ and $(2, 10)$

Word Problems: See Example on p. 244.

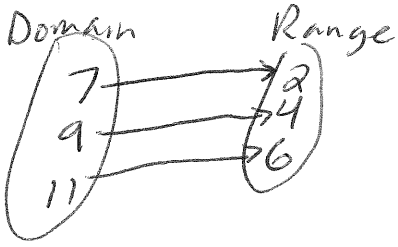
3.8 - Functions

Domain: values functions take in (x values)
Range: outputs of functions (y values)

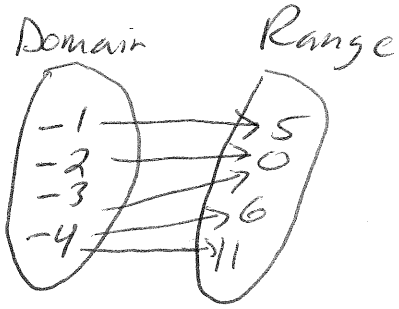
As data, $\{(1,7),(2,3),(3,8),(4,6)\}$ is a function
$\{(1,7),(2,3),(2,8),(4,16)\}$ is not a function

So $y = f(x)$ is a **function** of x given a relation for x and y if for each x there is **exactly** one y in the range.

Ex)



x	y
2	3
5	7
2	1
6	5

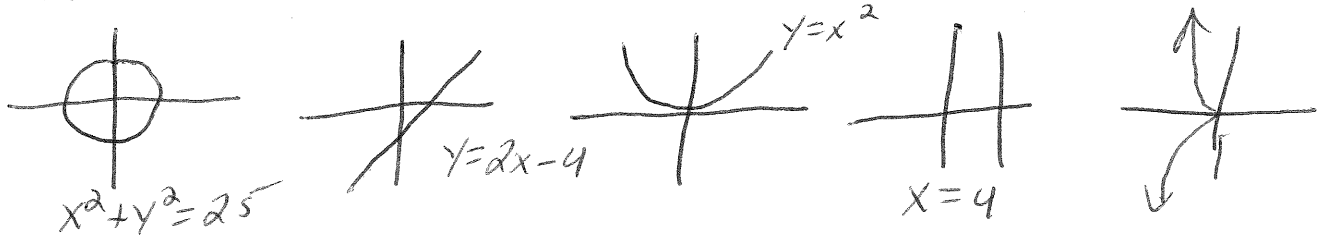


Ex) $f(x) = 5x + 7$ Find $f(4), f(-3), f(t)$

Ex) $g(x) = 3 - 2x$
$h(x) = x^3 + x^2 - 1$
$j(x) = x^4 + x^2 + 1$

Graph $f(x) = 4x + 1$
$f(x) = |x|$
$f(x) = |x| + 2$

Vertical Line Test: If a vertical line intersects a graph in more than one place, then the graph is not a function. Are these functions?



The function $C(h) = 80 + 15(h-4)$ gives cost to rent for an inflatable jumper for (4 hours min) h hours.

Find cost of 10hr rental?

12hr rental?

Section 4.1 - Systems of Equations by graphing.

Ex) Is $(-2, 5)$ a soln to $\begin{cases} 4y = 18 - x \\ y = 2x \end{cases}$

$\begin{cases} 3x + 2y = 4 \\ x - y = -7 \end{cases}$

Solve by Graphing:

$\begin{cases} 2x + 3y = 2 \\ 3x - 2y = 16 \end{cases}$ $\quad$ $\begin{cases} 2x - y = 5 \\ x + y = -1 \end{cases}$

$\begin{cases} y = -2x - 6 \\ 4x + 2y = 8 \end{cases}$ $\quad$ $\begin{cases} y = \frac{3}{2}x \\ 3x - 2y = 6 \end{cases}$

$\begin{cases} y = 2x + 4 \\ 4x + 8 = 2y \end{cases}$ $\quad$ $\begin{cases} 6x - 4 = 2y \\ y = 3x - 2 \end{cases}$

You can identify # of solutions without graphs

Same slopes, different intercepts $\Rightarrow$ No solutions

Same slopes, Same slopes $\Rightarrow$ Infinite solns

different slopes $\Rightarrow$ 1 solution.

Section 4.2 - Systems of Equations (Substitution)

For substitution, we solve for x or y in one equation. Then substitute the expression for x or y into the second equation to solve for one variable. Then plug the answer into either equation to find the second variable. Check your answer.

Ex) $\begin{cases} y = 3x - 2 \\ 2x + y = 8 \end{cases}$ $\begin{cases} x + 4y = 7 \\ x = 6y - 3 \end{cases}$

Variable is already found

$\begin{cases} 4x + 27 = 7y \\ x = -5y \end{cases}$ $\begin{cases} 3x + 40 = 8y \\ x = -4y \end{cases}$

$\begin{cases} 4x + y = 3 \\ 3x + 5y = 15 \end{cases}$ $\begin{cases} 2x - 3y = 10 \\ 3x + y = 15 \end{cases}$

$\begin{cases} 3a - 3b = 5 \\ 3 - a = 3b \end{cases}$ $\begin{cases} 2s - t = 4 \\ 3s - 5t = 2 \end{cases}$

$\begin{cases} \frac{y}{4} = \frac{-x}{2} - \frac{3}{4} \\ 2x - y = -1 + y - x \end{cases}$ $\begin{cases} \frac{y}{6} = \frac{x}{3} + \frac{1}{2} \\ 2x - y = -3 + y - x \end{cases}$

$\begin{cases} 4y - 12 = x \\ y = \frac{1}{4}x \end{cases}$ $\begin{cases} x - 4 = y \\ -2y = 4 - 2x \end{cases}$

$\begin{cases} x - 4 \neq y \\ -2y = 4 - 2x \end{cases}$ $\begin{cases} x = -3y + 6 \\ 2x + 6y = 12 \end{cases}$ $\begin{cases} y = 2 - x \\ 3x + 3y = 6 \end{cases}$

§4.3 - Systems of Equations (Elimination)

We will multiply one of the equations by a number (if necessary) so when we add the two equations, on variable is cancelled out, so we solve for the other. Then use either equation to find the second variable.

Ex) $\begin{cases} 2x+5y=11 \\ 6x-5y=13 \end{cases}$ $\begin{cases} -4x+3y=4 \\ 4x+5y=28 \end{cases}$

$\begin{cases} 2x+7y=-18 \\ 2x+3y=-10 \end{cases}$ $\begin{cases} 2x+7y=-27 \\ 3x+7y=-30 \end{cases}$

$\begin{cases} 7x+2y-14=0 \\ 9x=4y-28 \end{cases}$ $\begin{cases} 3x=10-2y \\ 5x-6y+30=0 \end{cases}$

$\begin{cases} 4a+7b=-8 \\ 5a+6b=1 \end{cases}$ $\begin{cases} 5a+3b=-7 \\ 3a+4b=9 \end{cases}$

$\begin{cases} \frac{1}{6}x+\frac{1}{2}y=\frac{1}{3} \\ -\frac{x}{9}+y=\frac{5}{9} \end{cases}$ $\begin{cases} -\frac{1}{5}x+y=\frac{8}{5} \\ \frac{x}{8}+\frac{y}{2}=\frac{1}{4} \end{cases}$

$\begin{cases} 3x-2y=2 \\ -3x+2y=-12 \end{cases}$ $\begin{cases} 2x-7y=5 \\ -2x+7y=3 \end{cases}$

$\begin{cases} \frac{2x-5y}{15}=\frac{8}{15} \\ -0.2x+.5y=-.8 \end{cases}$ $\begin{cases} \frac{3x+y}{6}=\frac{1}{3} \\ -0.3x-0.1y=0.2 \end{cases}$

Section 4.4 - Problem-Solving

Steps to solve problems

1. Write down given information
2. Assign variables (such as x and y) to represent the unknown quantities.
3. For the system of equations (the chart method can be very useful here)
4. Solve the system using substitution or elimination.
5. State solution with units.
6. Check to see if answers make sense (No - answers for lengths!)

Examples: See Ex 1 p. 315 → General Prob.
Ex 2 p. 316 → Geometry Prob
Ex 3 p. 317 → Perimeter Prob.
Ex 5 p. 319 → Interest Prob.
Ex 6 p. 320-1 → Distance Prob
Ex 7 p. 321-2 → Solution Prob
Ex 8 p. 322-3 → Mixture Prob.

4.5 - Solving Systems of Inequalities

Instead of solving a system of equations (= signs) we will have inequalities ($>, <, \geq, \leq$)

Steps for solving by graphing.

(Pretend it's $a =$)

① Draw the first line ($y = mx + b$ is easiest to use)
   Using solid line ——— for $\leq$ or $\geq$
   and dashed line - - - for $<$ or $>$

② Shade using one style for first line say
   ≡ or ||||

③ Repeat steps ① and ② for second line and use different shading than first line

④ Label solution set with S

⑤ Pick a test point to check answer.

Ex) $\begin{cases} x+y \geq -1 \\ x-y \geq 1 \end{cases}$ $\begin{cases} x-y \leq 2 \\ x+y \geq -1 \end{cases}$ $\begin{cases} y > 3x \\ 2x+y < 4 \end{cases}$

$\begin{cases} x+3y < 3 \\ y > \frac{1}{3}x \end{cases}$ $\begin{cases} x \leq 2 \\ y > 3 \end{cases}$ $\begin{cases} y \leq 1 \\ x > 2 \end{cases}$

$\begin{cases} x \geq 0 \\ y \geq 0 \\ x+2y \leq 6 \end{cases}$ $\begin{cases} x \leq 1 \\ y \leq 2 \\ 2x-y \leq 4 \end{cases}$

Section 5.1 - Exponent Rules

$x^n = \underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}_{n\text{-times}}$ $\quad 3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Ex) $(-2x)^3$ Base $= -2x$, Exp $= 3$

Ex) 5^4 Base: 5, Exp $= 4$

Ex) t^{10} Base: t, Exp: 10

Ex) $5 \cdot t \cdot t \cdot t$

Ex) $6^2 \cdot 6^3$

Ex) $9^5(9^6)$

Rule: $x^m \cdot x^n = x^{m+n}$

Rule: $\frac{x^m}{x^n} = x^{m-n}$

Ex) $\frac{4^5}{4^2}$

Ex) $\frac{x^9}{x^3}$

Ex) $\frac{a^3 a^5 a^7}{a^4 a^2}$

Rule: $(x^m)^n = x^{mn}$

Ex) $(3^2)^4$

Ex) $(z^2)^3$

Ex) $((-6)^2)^5$

Rule: $(xy)^n = x^n y^n$

Rule: $x^1 = x$

Rule: $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

Ex) $(3c)^3$

Ex) $(x^2 y^3)^3$

Ex) $\left(-\frac{1}{4} a^3 b\right)^2$

Ex) $\left(\frac{x}{7}\right)^3$, $\left(\frac{2x^3}{3y^2}\right)^4$

Ex)

Section 5.2 - Zero and Negative Exponents

Rules $x^0 = 1$ for any $x \neq 0$

$x^{-n} = \frac{1}{x^n}$ for $x \neq 0$

$\frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}$ for $x, y \neq 0$

Rules $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$

Ex) 8^{-2}, x^{-5}, $(-3)^{-3}$, 2^{-3}, 4^0

Ex) $9m^{-3}$, -5^{-2}, $12h^{-9}$, -2^{-4}

Ex) $\frac{1}{d^{-10}}$, $\frac{2^{-3}}{2^{-4}}$, $\frac{-6s^{-2}}{t^{-9}}$, $\frac{1}{w^{-5}}$, $\frac{-8h^{-6}}{a^{-7}}$

Ex) $\left(\frac{4}{m}\right)^{-2}$ $\left(\frac{c}{9}\right)^{-2}$

Ex) $x^5 \cdot x^{-3}$, $(2a^3b^{-5})^3$, $(x^3)^{-2}$, $\left(\frac{c}{2}^4\right)^{-3}$

$(4c^2d^{-1})^3$, $\left(\frac{k^6}{6}\right)^{-3}$, $(5g^3h^{-1})^3$

Ex) $\frac{y^{-4}y^{-3}}{y^{-20}}$, $\frac{7^{-1}a^3b^4}{6^{-2}a^5b^2}$, $\left(\frac{x^{-3}y^2}{xy^{-3}}\right)^2$

Ex) $\frac{a^{-4}a^{-5}}{a^{-3}}$, $\frac{1^{-4}(x^5)(y^3)}{9^{-2}x^3y^6}$ $\left(\frac{c^{-2}d^2}{c^4d^{-3}}\right)^3$

Section 5.3 - Scientific Notation

Power of 10	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Value	10,000	1,000	100	10	1	$\frac{1}{10} = .1$	$\frac{1}{100} = .01$	$\frac{1}{1000} = .001$

Ex) 3.67×10^2 Ex) 2.158×10^{-3} Ex) 4.0×10^{57}

Ex) 150,000,000 Ex) 0.00000256 Ex) 432×10^5

Always $A \times 10^b$ A is in $[1, 10)$, b an integer

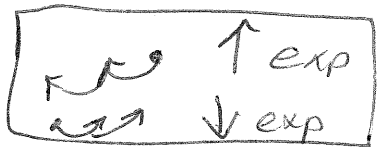
Ex.) .00009055, Ex) 88×10^{-3}, Ex) 9000,000

Light from Sirius reaches Earth in 70,000 hours
If light travels 670,000,000 mph, How far from Earth is Sirius?

Ex) Use Sci Notation to do $(2{,}540{,}000{,}000{,}000)(.00041)$
Ans: 1.0414×10^9

Rules: $(a \times 10^m)(b \times 10^n) = (a \cdot b) \times 10^{m+n}$

$\frac{(a \times 10^m)}{(b \times 10^n)} = \left(\frac{a}{b}\right) \times 10^{m-n}$



Ex) $(3 \times 10^5)(4 \times 10^6)$

Section 5.4 - Polynomials

A **polynomial** is a sum of terms in which all variables have **whole number exponents**.

Ex) $3x-2$ $\quad ax^2+bx+c$

x^2+3x+3

Monomial	Binomial	Trinomial
$6x$	$9x-3$	$5t^2-3t+1$
$5x^3y^2$	72^4-2^2	x^3+x^2+x
11	$18a^2b+4ab$	$\frac{1}{2}x^2+x+1$

The **degree** of a polynomial is the value of the highest exponent of the variable

Ex) x^2+x+1 degree 2 $\quad (x+5)^8$ degree 8

x^3+1 degree 3 $\quad (x+2)(x-1)^3$ degree 4

Evaluate: $3x^2+4x-5$ for $x=0$, $x=-2$

$3p^2q-4pq^2$ for $p=2$, $q=-3$

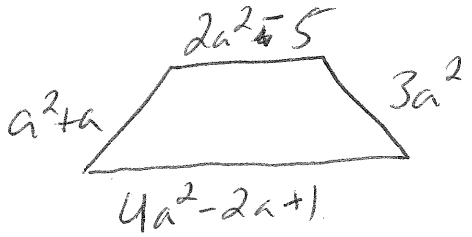
Graph: $y=x^2$, $y=x^2-2$ (by table), $y=x^3+1$, $y=x^3$

Section 5.5 - Adding / Subtracting Polynomials

Ex) $(3x^2+6x+7)+(2x-5)$

$(-6a^3+5a^2-7a+9)+(4a^3-5a^2-a-8)$

Find a polynomial that represents the perimeter of the trapezoid



Ex) $4x^2-3$ and $3x^2-8x+8$ in vertical form.

$4q^2-7$ and $2q^2-8q+9$ " "

Ex) $-(2a^2-a+9)$

$(3a^2-4a-6)-(2a^2-a+9)$

$(8a^3-5a^2+5)-(a^3-a^2-7)$

$(x^2y-2x+y-2)-(6x+9y-2)$

Ex) $3x^2-2x$ from $2x^2+4x+1$

$4x^3-6x^2+x$ from $7x^3-2x$

$3x^8-2xy^3-4x$ from $3x-4xy^3+10x^8$

Fraction Examples

Section 5.6 – Multiplying Polynomials

Ex) $6r \cdot r$, $3t^4(-2t^5)$, $\left(\frac{1}{3}a^2b^3\right)(21ab^2)$

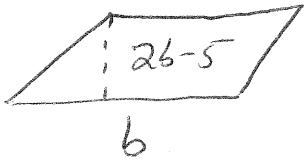
$-4y^5z^2(2y^3z^3)(3yz)$

Ex) $3n^6(16n^{15}+n^{10})$

$-2xz^3(6x^3z+x^2z^2-xz^3+7z^4)$

$3a^2(3a^2-5a+2)$

Find a polynomial that gives area of the parallelogram



Binomials

$(2a+4)(3a+5)$

$(5x-8)(x+1)$

FOIL

First, Outer, Inner, Last

$(3x+1)(2x-3)$

Ex) $(2r-\frac{1}{2})(2r+\frac{5}{2})$

$(x+5)(x+7)$

$(5az+3bz)(4a+3b)$

$(3x+4)(2x-3)$

$(3a^2-7b)(a^2-b^2)$

Ex) $(7y+3)(6y^2-8y+1)$

$(3a^2-1)(2a^4-a^2-a)$

$-3a(4a+1)(a+1)$

$(3a^2-4a+7)(2a+5)$

$(6y^3-5y+4)(-4y^2-3)$

$-2y(y+3)(3y-2)$

Section 5.7 - Special Product

Square a binomial: $(x+y)(x+y)$
$(x-y)(x-y)$

Ex) $(t+9)^2$, $(8a-5)^2$, $\left(c^3-\frac{7}{2}d\right)^2$

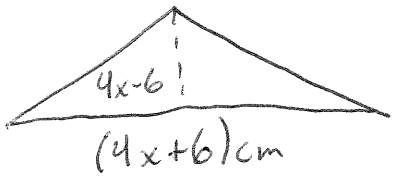
Sum and Difference: $(x-y)(x+y)$

Ex) $(m+2)(m-2)$ $(3y+4)(3y-4)$ $\left(b-\frac{2}{3}\right)\left(b+\frac{2}{3}\right)$

Higher Degrees: $(x+1)^3$, $(y+1)^4$ (Pascal's Triangle)

Simplify: $-8(y^2-2y+3)-4(2y^2+y-6)$
$(x-1)(x-2)+3x(x+3)$
$(3y-2)^2-(y-5)(y+5)$

Ex) Find a polynomial that represents the area of the triangle



$A=\frac{1}{2}bh$

Ex) $(t^2+u^2)^2$

Section 5.8 - Dividing Polynomials

Ex) $\frac{21x^5}{x^2}$, $\frac{10r^6s}{6rs^3}$

Rule: $\frac{a}{d}+\frac{b}{d}=\frac{a+b}{d}$

Ex) $\frac{9x^2+3x}{3x}$ $\qquad$ $\frac{12a^4b^3-18a^3b^2+2a^2}{6a^2b^2}$

Polynomial Division (Long Division)

Ex) ⊛ $(x^2+5x+6)\div(x+2)$

⊛ $(x^2+7x+12)\div(x+3)$

$(6x^2-7x-2)\div(2x-1)$

$(27x^3+1)\div(3x+1)$

$(8x^2+6x-3)\div(2x+3)$

⊛ $(4x^2+2x^3+12-2x)\div(x+3)$

$(6x^3+x^2-10x+4)\div(2x-1)$

⊛ $(x^4+x^3+x^2+x+1)\div(x+1)$

Synthetic Division

Ex) Try the above examples above with ⊛

* Remember to switch sign for number in box.

Section 6.1 - Factoring by Grouping

Ex) $8m+24$ $\quad 6f+36$

$35a^3b^2-14a^2b^3$ $\quad 24s^2t^2-42s^3t$

$3x^4-5x^3+x^2$ $\quad y^6-10y^4-y^3$

Ex) Factor -1 from each

$-a^3+2a^2-4$, $6-x$, $-b^4-3b^2+2$

Ex) Factor $-20m+30$, $-44c+55$

Ex) Factor $x(x+4)+3(x+4)$

$2y(y-1)+7(y-1)$

Ex) Factor by grouping $2x^3+x^2+12x+6$

$5c-5d+cd-d^2$

$4t+4s+4tz+4sz$ $\quad 3n^3+2n^2+9n+6$

$10k+10m-2km-2m^2$ $\quad 7x+7y+xy-y^2$

$x^2-ax-x+a$

6.2 - Factoring x^2+6x+C

"c"

Factors	Sum

Ex) $x^2+8x+15$

$y^2+7y+10$

$y^2-13y+12$

$g^2-2g-24$

p^2-6p+8

x^2+x-20

$z^2-4z-21$

$-h^2+2h+63$

Ex) $x^2-4xy-5y^2$

$s^2+6st-7t^2$

Ex) $2x^4+26x^3+80x^2$

$4m^5+8m^4-32m^3$

$-13g^2+36g+g^3$

t^3+4t^2-12t

Ex) (Non Factorable)

x^2+2x+3

x^2-4x+6

Ex) x^2+x-20

$x^2+5x-4x-20$

$x^2-4xy-5y^2$

$x^2-5xy+xy-5y^2$

Ex) $2x^3-20x^2-18x$

$2x(x^2-10x+9)$

$(-9x-1x)$

Ex) $3m^3-27m^2+24m$

Section 6.3 - Factoring ax^2+bx+c

Ex) $2x^2+5x+3$

$2x^2+5x+2$

$6a^2-17a+5$

$6b^2-19b+3$

Ex) $3y^2-7y-6$

$5t^2-23t-10$

Ex) $4b^2+8bc-45c^2$

$4x^2+4xy-3y^2$

Ex) $-8x^3+2x^2+3x$

$-14y^3+22y^2+12y$

Ex) $10x^2+13x-3$

$15a^5-17x^4+6x^3$

Ex) $21a^4-13a^3+2a^2$

Section 6.4 - Factoring Perfect Square / Diff of 2 squares

Recall: $A^2+2AB+B^2=(A+B)^2$

$A^2-2AB+B^2=(A-B)^2$

Ex) $x^2+20x+100$

$9x^2-30xy+25y^2$

Ex) $4a^3-4a^2+a$

$49x^3-14x^2+x$

Recall: $(A+B)(A-B)=A^2-B^2$

Ex) x^2-9 n^2-45 (*)

$16-b^2$ a^2-81

Ex) c^2-4 s^2+36 (*)

x^2-24 (*) $121-t^2$

Ex) $25x^2-49$ $16y^2-9$

$-121z^2+4y^4$ $9m^2-64n^4$

Ex) $8x^2-8$

x^4-16

Section 6.5 - Sum/Difference of Cubes

Rules: $F^3+L^3 = (F+L)(F^2-FL+L^2)$

$F^3-L^3 = (F-L)(F^2+FL+L^2)$

Ex) x^3-8

h^3+27

Ex) a^3-64b^3

$8c^3-1$

Ex) $-2t^5+250t^2$

$4c^3+4d^3$

Section 6.6 - Factoring Strategy

Steps (1) Factor out common terms from each piece

(2) a) 2 terms
* Try special formulas * groupings

b) 3 terms
* Try special formulas
* Try grouping

c) 4+ terms
* Try groupings

(3) Factor remaining terms

(4) Check factorization

Ex) $2x^4-162$

$11a^6-11a^2$

$-4c^5d^2-12c^4d^3-9c^3d^4$

$-32h^4-80h^3-50h^2$

Ex) y^4-3y^3+y-3

5^4+6^3+86+8

$4n^3-4n^2+32n$

$6m^2-54m+6m^3$

Ex) $3y^3-4y^2-4y$

$6y^3+21y^2-12y$

Section 6.7 - Solving Quadratics by Factoring

Ex) $(4x-1)(x+6)=0$

Defn: A quadratic equation has the form

$$ax^2+bx+c=0 \qquad a\neq 0$$

Defn: The zero property states that if a and b are expressions (or numbers), and

$a \cdot b = 0$, then $a=0$ or $b=0$

Ex) $(x-12)(5x+6)=0$

$x^2-2x-63=0$

$x^2+5x+6=0$

Ex) $x^2-25=0$

$x^2-49=0$

Ex) $6x^2=12x$

$5x^2=25x$

Ex) $2x^2-2=-3x$ $(-2, \frac{1}{2})$

$3x^2-8=-10x$ $(\frac{2}{3}, -4)$

Ex) $x(9x-12)=-4$

$x(4x+12)=-9$

Ex) $6x^3+12x=17x^2$

$10x^3+\blacksquare x^2=2x$

Practice

(1) $4a^2+1=8a+1$

(2) $x^3-6x^2=-9x$

(3) $5f(5f-16)=-15$

Section 6.8 - Applications of Quadratics

Ex) p. 490, Ex 1
p. 491, Ex 2
p. 493, Ex 4
p. 494, Ex 5

Section 7.1 - Simplifying Rational Expressions

A rational expression is of the form $\frac{A}{B}$ where A and B are ~~polynomials~~ polynomials and $B \neq 0$.

Ex) Evaluate $\frac{2x-1}{x^2+1}$ for $x=-3$, $x=0$

Find where the expressions are undefined.

a) $\frac{7x}{x-5}$ b) $\frac{3x-2}{x^2-x-6}$ c) $\frac{8}{x^2+1}$ d) $\frac{x+5}{12}$

Rule: $\frac{AC}{BC} = \frac{A}{B}$ $B, C \neq 0$

Simplify:

Ex) $\frac{21x^3}{14x^2}$ Ex) $\frac{30t-6}{36}$ Ex) $\frac{x^2+13x+12}{x^2+12x}$

Ex) $\frac{x^3+x^2}{x+1}$

Simplify

Ex) $\frac{4t-20}{12}$ Ex) $\frac{x^2-6-x}{x^2-3x}$ Ex) $\frac{2x^4+4x^3}{x+2}$

Simplify

Ex) $\frac{3x^2-8x-3}{2x^5-18x^3}$ Ex) $\frac{(x-y)^4}{x^2-2xy+y^2}$

Ex) $\frac{4x^2-4x-15}{8x^3-50x}$ Ex) $\frac{(a+3b)^5}{a^2+6ab+9b^2}$

Simplify

Ex) $\frac{5(x+3)-5}{7(x+3)-7}$ Ex) $\frac{4(x-2)+4}{3(x-2)+3}$

Ex) $\frac{2a-1}{1-2a}$ Ex) $\frac{3p-2}{2-3p}$

Ex) $\frac{y^2-1}{3-3y}$ Ex) $\frac{t+8}{t-8}$

Section 7.2 - Multiplying / Dividing Rational Exp.

Rule: $\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$ Rule: $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}$

or $\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}$

Ex) $\frac{x+1}{x} \cdot \frac{9}{4x^2}$ $\frac{35x^3}{17y} \cdot \frac{y}{5x}$

Ex) $\frac{x+3}{2x+4} \cdot \frac{6}{x^2-9}$ $\frac{8x^2-8x}{x^2+x-56} \cdot \frac{3x^2-22x+7}{x-x^2}$

Ex) $\frac{3n-9}{3n+2} \cdot \frac{9n^2-4}{6}$ $\frac{m^2-4m-5}{2m-m^2} \cdot \frac{2m^2-4m}{3m^2-14m-5}$

Ex) $63x\left(\frac{1}{7x}\right)$ Ex) $5a\left(\frac{3a-1}{a}\right)$

Division

Ex) $\frac{a}{13} \div \frac{17}{26}$ Ex) $\frac{9x}{35y} \div \frac{15x^2}{14}$

Ex) $\frac{8a}{3b} \div \frac{16a^2}{9b^2}$

Ex) $\frac{x^2+x}{3x-15} \div \frac{(x+1)^2}{6x-30}$

Ex) $\frac{z^2-9}{z^2+4z+3} \div \frac{z^2-3z}{(z+1)^2}$

Ex) $\frac{2x^2-3xy-2y^2}{2x+y} \div (4y^2-x^2)$

Ex) $(b-a) \div \frac{a^2-b^2}{a^2+ab}$

Ex 1) A roll of carpet is 12 ft wide, 150 ft long
Find # of sq. yd. of carpeting

Conversion: $1yd^2 = 9ft^2$

$$\frac{1800\ ft^2}{1\ roll} = \frac{1800\ ft^2}{1\ roll} \cdot \frac{1yd}{9\ ft^2} = \frac{200yd^2}{1\ roll}$$

Ex 2) Convert speed of light $186{,}000 \frac{mi}{s}$ to $\frac{mi}{min}$

$$\frac{186{,}000\ mi}{1\ s} \cdot \frac{60\ s}{1\ min} = \frac{11{,}160{,}000\ mi}{1\ min}$$

Section 7.3 - Add/Sub. with Like Denom. (Least Common) ①

Rules: $\frac{A}{D}+\frac{B}{D}=\frac{A+B}{D}$ $\quad\frac{A}{D}-\frac{B}{D}=\frac{A-B}{D}$

Add)

$\frac{x}{8}+\frac{3x}{8}$

$\frac{4s-9}{9t}+\frac{7}{9t}$

$\frac{2x}{15}+\frac{4x}{15}$

$\frac{3n-8}{23n}+\frac{2}{23n}$

$\frac{3x+21}{5x+10}+\frac{8x+1}{5x+10}$

$\frac{x^2+9x-7}{2x(x-6)}+\frac{x^2-9x}{2x(x-6)}$

$\frac{c^2-c}{(c-1)(c+2)}+\frac{c^2-10c}{(c+2)(c-1)}$

Subtract)

$\frac{x+6}{x^2+4x-5}-\frac{1}{x^2+4x-5}$

$\frac{n-3}{n^2-16}-\frac{1}{n^2-16}$

$\frac{3y^2}{(y+3)(y-3)}-\frac{-3y^2+y}{(y+3)(y-3)}$

$\frac{x^2+10x}{x+3}-\frac{4x-9}{x+3}$

$\frac{x^2}{(x+7)(x-8)}-\frac{-x^2+14x}{(x+7)(x-8)}$

$\frac{x^2+3x}{x-1}-\frac{5x-1}{x-1}$

Least Common Denominator

LCD of $\frac{11}{8x}$ and $\frac{7}{18x^2}$

$72x^2$

$\frac{20}{x}$ and $\frac{4x}{x-1}$

Find LCD: $\frac{x}{7x+7}$ and $\frac{x-2}{5x+5}$; $\frac{6-x}{x^2+8x+6}$; $\frac{15x}{x^2-16}$

Write $\frac{x+1}{x^2+6x}$ with denominator $x(x+6)(x+2)$

$\frac{x-3}{x^2-4x}$ " " $x(x-4)(x+8)$

Section 7.4 - Add/Subtract (Not same denominator)

Add) $\frac{9x}{7}+\frac{3x}{5}$ $\frac{13}{18b^2}-\frac{1}{24b}$

$\frac{y}{2}+\frac{6y}{7}$ $\frac{5}{21z^2}-\frac{3}{28z}$

Add: $\frac{3}{2x+18}+\frac{27}{x^2-81}$ $\frac{x}{x-1}-\frac{x-6}{x-4}$

$\frac{2}{5x+25}+\frac{4}{x^2-25}$ $\frac{x}{x+9}-\frac{x-7}{x+8}$

Subtract: $\frac{m}{m^2+5m+6}-\frac{2}{m^2+3m+2}$ $\frac{4b}{a-5}+b$

$\frac{6}{b^2-2b-8}-\frac{6}{b^2+b-20}$ $\frac{10y}{n+4}+y$

Subtract: $\frac{x}{x-7}-\frac{1}{7-x}$ $\frac{n}{n-8}-\frac{12}{8-n}$

Section 7.5 Complex Fractions

Simplify: $\dfrac{\frac{5x^2}{3}}{\frac{2x^3}{9}}$ $\dfrac{\frac{7y^3}{8}}{\frac{21y^2}{20}}$

Simplify: $\dfrac{\frac{1}{2}-\frac{1}{x}}{\frac{x}{3}+\frac{1}{5}}$ $\dfrac{\frac{1}{3}+\frac{1}{x}}{\frac{x}{5}-\frac{1}{2}}$

Simplify: $\dfrac{\frac{6}{x}+y}{\frac{6}{y}+x}$ $\dfrac{\frac{2}{a}-6}{\frac{2}{6}-a}$

Simplify: $\dfrac{\frac{1}{2}-\frac{1}{x}}{\frac{x}{3}+\frac{1}{5}}$ $\dfrac{\frac{1}{4}-\frac{1}{x}}{\frac{x}{5}+\frac{1}{3}}$

Simplify: $\dfrac{\frac{1}{8}-\frac{1}{y}}{\frac{8-y}{4y^2}}$ $\dfrac{\frac{10-n}{5n^2}}{\frac{1}{10}-\frac{1}{n}}$

Simplify: $\dfrac{1}{1+\frac{1}{x+1}}$ $\dfrac{2}{\frac{1}{x+2}+2}$

Section 7.6 - Solving Rational Equations

Solve! $\frac{2x}{3} = \frac{x}{6} + \frac{3}{2}$

$\frac{3x}{5} = \frac{x}{2} + \frac{1}{10}$

$\frac{2}{x} + \frac{1}{4} = \frac{5}{2x}$

$\frac{1}{6} + \frac{4}{3x} = \frac{5}{x}$

Solve: $y - \frac{12}{y} = 4$

$x - \frac{24}{x} = -5$

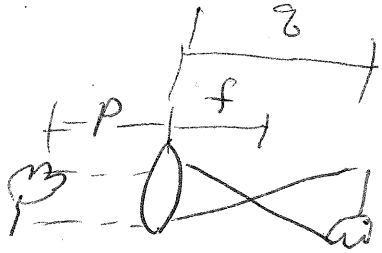
$\frac{11x}{x-5} = 6 + \frac{55}{x-5}$

$\frac{9x}{x-6} = 3 + \frac{54}{x-6}$

Solve $\frac{x+5}{x+3} + \frac{1}{x^2+2x-3} = 1$

$\frac{1}{x+3} + \frac{1}{x-3} = \frac{5}{x^2-9}$

Ex) Camera lenses follow $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$



Solve for q.

Solve for p.

Section 7.7 - Word Problems / Rational Expressions

p567, Ex 1
p567, Ex 2
p.569, Ex 3
p.570, Ex 4
p572, Ex 5

Section 7.8 - Proportions / Similar Triangles

A ratio is a quotient of numbers/quant with same units
A rate is " " " with different "
A proportion states two ratios or two rates are equal

Ratio

Ex) Ratio of 5 to 9 $\Rightarrow \frac{5}{9}$

" 12oz to 2lbs

$\frac{12oz}{32oz} = \frac{3 \cdot 4}{4 \cdot 8} = \frac{3}{8}$

Rate

Ex) $\frac{495 mi}{9 hours} = 55 mi/hr$

$\frac{16 m}{2 s} = 8 \frac{m}{s}$

Proportion

Ex) $\frac{3 waiters}{7 tables} = \frac{9 waiters}{21 tables}$

Rules

$\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$

$ad = bc \Rightarrow \frac{a}{b} = \frac{c}{d}$

Solve $\frac{3}{2} = \frac{9}{x}$ $\qquad$ $\frac{a}{2} = \frac{4}{a-2}$

$\frac{15}{x} = \frac{25}{40}$ $\qquad$ $\frac{6}{c} = \frac{c-1}{5}$

Ex) Ex 5, p 580

Ex 7, p 581-2

Two triangles are <u>similar</u> if sides are <u>proportional</u>

Ex 8, p 582-3