MATH 410 MIDTERM 2 April 27, 2016

Name: KEY Score: _____ / 100

Student ID: _____

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	10	11	12	Total
✓													
Score													

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 11 question exam (One extra credit problem can be attempted for a total of 12 questions).
- Students have 2 hours and 15 minutes to complete the exam.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. Clearly box your final answer.
- You **MUST** complete **11** problems for credit. In the above table in the row with the ✓, please mark with a ✓ which problems you want to be graded. If you wish to do a 12^{th} problem for extra credit, please write *EC* in the ✓ row for the problem you wish to be counted for extra credit.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- Each of the 11 questions you choose to do will be graded out of 3 points. The score will then be totaled and multiplied by 3 to get a raw score out of 99 points. One point will be given for clearly writing your name on the exam sheet. This will get you to 100 points. If you choose to do a 12^{th} problem for extra credit, the most that will be awarded for that question will be 3 points. So, the highest possible score on this examination is 103 points out of 100.
- The back of the test can be used for scratch work.

GOOD LUCK!

FORMULAS:

Name	Formula
Square of Sum	$(A+B)^2 = A^2 + 2AB + B^2$
Square of Difference	$(A-B)^2 = A^2 - 2AB + B^2$
Sum of Square	$(A+B)(A-B) = A^2 - B^2$
Difference of Cubes	$F^3 - L^3 = (F-L)(F^2 + FL + L^2)$
Sum of Cubes	$F^3 + L^3 = (F+L)(F^2 - FL + L^2)$

1) Use the rules of exponents to simplify the expression, do not use negative exponents:

$$\frac{(3x^5y^3)^3}{18x^{-15}y^{12}}$$

$$= \frac{3^3(x^5)^3(y^3)^3}{18x^{-15}y^{12}} = \frac{27x^{15}y^9}{18x^{-15}y^{12}}$$

$$= \frac{3}{2}x^{15-(-15)}y^{9-12}$$

$$= \frac{3}{2}x^{30}y^{-3} = \boxed{\frac{3}{2}\frac{x^{30}}{y^3}}$$

2) Use the rules of exponents to simplify the expression, do not use negative exponents:

$$\frac{4^{-3}a^{-7}b^4}{2^{-3}a^7b^8}$$

$$= \frac{2^3a^{-7}b^4}{4^3a^7b^8} = \frac{8}{64}a^{-7-7}b^{4-8}$$

$$= \frac{1}{8}a^{-14}b^{-4}$$

$$= \boxed{\frac{1}{8a^{14}b^4}}$$

3) Do the following operations (answers must be in standard form for scientific notation!):

a) $(2 \times 10^{-4}) \cdot (4 \times 10^{-2})$

b) $(50 \times 10^{-5}) \div (2 \times 10^{5})$

c) $(8 \times 10^{4}) \cdot (6 \times 10^{6})$

a) $(2\times10^{-4})\cdot(4\times10^{-2}) = (4\cdot2)\times10^{-4-2} = \boxed{8\times10^{-6}}$

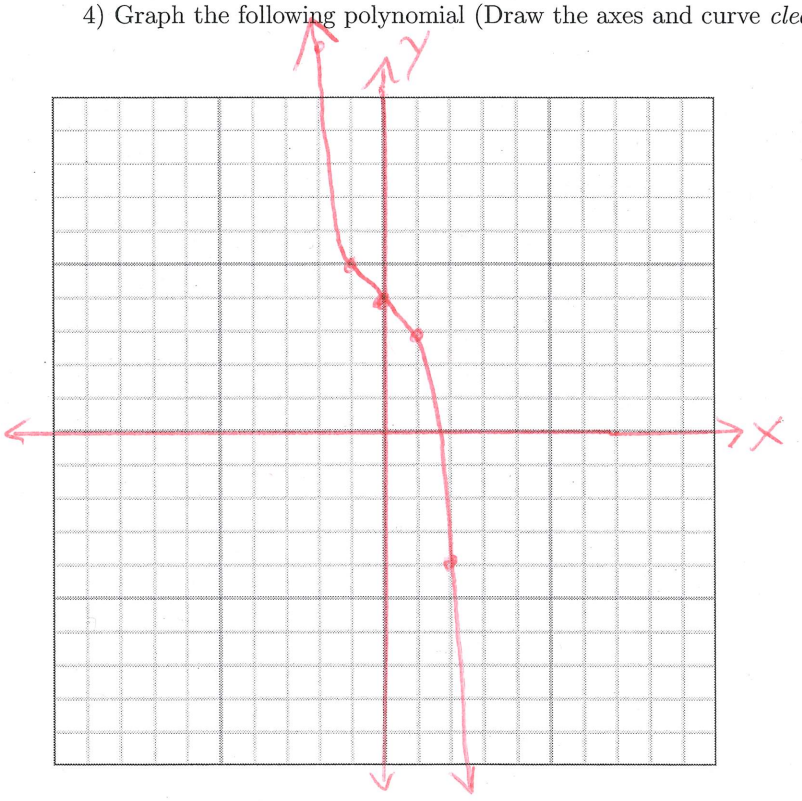
b) $\dfrac{(50\times10^{-5})}{(2\times10^{5})} = \left(\dfrac{50}{2}\right)\times10^{-5-5} = 25\times10^{-10}$

$= \boxed{2.5\times10^{-9}}$

c) $(8\times10^{4})\cdot(6\times10^{6}) = (8\cdot6)\times10^{6+4} = 48\times10^{10}$

$\boxed{4.8\times10^{11}}$

4) Graph the following polynomial (Draw the axes and curve *clearly*): $f(x) = -x^3 + 4$



x	y
-2	12
-1	5
0	4
1	3
2	-4
3	-23

5) Expand the following binomial $(4x^2 + y^2)^2$

Method 1

$(4x^2+y^2) = (4x^2+y^2)(4x^2+y^2)$

$= 16x^4 + 4x^2y^2 + 4x^2y^2 + y^4$

$= \boxed{16x^4 + 8x^2y^2 + y^4}$

$(4x^2+y^2)^2 = (4x^2)^2 + 2(4x^2)(y^2) + (y^2)^2 = \boxed{16x^4+8x^2y^2+y^4}$

$(A+B)^2 = A^2 + 2AB + B^2$

Method 2

6) Perform the following divsion using either polynomial divsion or synthetic divsion:

$$\frac{x^4 - 5x^3 + 7x^2 - 5x + 6}{x - 3}$$

Synthetic division

$$\begin{array}{c|ccccc} 3 & 1 & -5 & 7 & -5 & 6 \\ & \downarrow & 3 & -6 & 3 & -6 \\ \hline & 1 & -2 & 1 & -2 & 0 \end{array}$$

$\boxed{x^3 - 2x^2 + x - 2}$

Polynomial Division

$$\begin{array}{r} x^3 - 2x^2 + x - 2 \\ x-3 \overline{)\, x^4 - 5x^3 + 7x^2 - 5x + 6} \\ -(x^4 - 3x^3) \\ \hline -2x^3 + 7x^2 - 5x + 6 \\ -(-2x^3 + 6x^2) \\ \hline x^2 - 5x + 6 \\ -(x^2 - 3x) \\ \hline -2x + 6 \\ -(-2x + 6) \\ \hline 0 \end{array}$$

$\boxed{x^3 - 2x^2 + x - 2}$

7) Factor the equation completely: $x^2 - bx - x + b$

Factor by grouping

$x^2 - bx - x + b$

$= x(x-b) - (x-b)$

$= \boxed{(x-b)(x-1)}$

8) Factor the equation completely: $3a^2 - 6a - 45$
(Do not use quadratic formula)

$3a^2 - 6a - 45 = 3(a^2 - 2a - 15)$

$= \boxed{3(a-5)(a+3)}$

9) Solve the following equation for x by factoring the left hand side: $6x^2 + 11x - 7 = 0$
(Do not use quadratic formula)

Two numbers adding to 11
Multiplying to $(6)(-7) = -42$

Factors	Sum
1, -42	-41
-1, 42	41
2, -21	-19
-2, 21	19
3, -14	-11
-3, 14	11
6, -7	-1
-6, 7	1

$\Rightarrow 6x^2 + 11x - 7 = 0$

$6x^2 - 3x + 14x - 7 = 0$

$3x(2x-1) + 7(2x-1) = 0$

$(3x+7)(2x-1) = 0$

$\boxed{x = \frac{1}{2}} \quad \boxed{x = -\frac{7}{3}}$

10) Factor the equation completely: $27y^3 - 1$

$F^3 = 27y^3 \Rightarrow F = 3y$

$L^3 = 1 \Rightarrow L = 1$

$\Rightarrow 27y^3 - 1 = \boxed{(3y-1)(9y^2+3y+1)}$

11) Solve the following equation for x: $2x^2 - 3 = 5x + 9$
(Do not use quadratic formula)

$2x^2 - 3 = 5x + 9$

$-5x-9 \quad -5x-9$

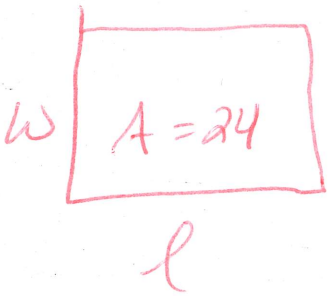
$2x^2 - 5x - 12 = 0$

$(2x+3)(x-4) = 0$

$2x+3=0 \qquad x-4=0$

$\boxed{x = -\frac{3}{2}} \qquad \boxed{x = 4}$

12) The area of a rectangular field is 24 km^2. The difference between the length and the width is 10 km. Find the dimensions of the field (the length and width). Label the units for the length and width. You must show work by defining functions for area, length and/or width in order to receive full credit.



$\left.\begin{aligned} A &= \ell w \\ \ell - w &= 10 \end{aligned}\right\} \Rightarrow$

$w(w+10) = 24$

$w^2 + 10w - 24 = 0$

$(w+12)(w-2) = 0$

$w+12=0 \qquad w-2=0$

$\cancel{w = -12} \qquad w = 2$

$\ell - w = 10$

$\ell - 2 = 10$

$\ell = 12$

$\Rightarrow \boxed{\begin{aligned} \ell &= 12 \text{ km} \\ w &= 2 \text{ km} \end{aligned}}$

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST