§1.9 - **Simplifying Algebraic Expressions**

Commutative Property: $a + b = b + a$ and $a \cdot b = b \cdot a$
Associative Property: $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Distributive Property: $a \cdot (b + c) = a \cdot b + a \cdot c$

Example 0.1. *Simplify the following:*
a) $-9 \cdot 3b$
b) $15a(6)$
c) $4b(5a)$
d) $\frac{15}{4} \cdot \frac{12}{5} r$
e) $-6(2y - 2)$
f) $3(-3y^2 - 4)$
g) $\frac{2}{3}(3x + 9y)$
h) $-(6y - 3)$
g) $-\frac{3}{4}(4x - \frac{8}{12} y)$

Combining like terms: Two expressions that look similar can be combined together. For example, $3x+5x$ can be combined to give $8x$. On the other hand, things like $3x + 4y$ cannot be combined because x and y are different variables. Therefore, we leave it as $3x + 4y$.

Example 0.2. *Simplify the following:*
a) $3x + 6y + 5x + 10y - x$
b) $2x + y - 2z - 5x + 6y + 3z$
c) $4x + 6y - 3z^2 + 2z - 6x - 2y + z^2$
d) $5x^2 - 6y^2 - 10x^2 - 4x - 2y - 3 + 10$
e) $4x^2 - z^3 + 10z^2 - 4x + 10 - 3x + 11$
f) $4(z^2 + 1)^2 + 3(x - 1)^2 - 6y - 4(x - 1)^2 + 2(z^2 + 1)$

§2.1 - **Solving Equations**

Example 0.3. *Solve the following for x:*
a) $x - 10 = 2$
b) $x + 5 = -2$
c) $3x - 5 = 7$
d) $x + \frac{1}{8} = \frac{7}{4}$
e) $\frac{x}{4} = 4$
f) $\frac{x}{4} = \frac{6}{12}$
g) $-\frac{5x}{4} = \frac{3}{16}$
h) $-\frac{6x}{5} = \frac{10}{3}$

<u>Practice Problems</u>
1) Simplify: $4z^2 + 3x - 4y + 3t^2 + 4x - z^2 - t^2 + 4 + 10 + (3t - 1)^2$
2) Solve for x: $2x + 5 - 3x + 2 = 8$
3) Solve for x: $-\frac{6}{5}(x - 5) = -5$

§2.2 - More on Solving Equations

We will need the following definitions about linear equations later, so we will give them now:
A linear equation can be written in standard form as: $ax + by = c$
A linear equation can be written in point-slope form as: $y - y_1 = m(x - x_1)$
A linear equation can be written in slope-intercept form as: $y = mx + b$

Solve:

Example 0.4. *a)* $-12x + 5 = 17$
b) $\frac{3}{4}x - 1 = 5$
c) $-0.2 = -0.8 - y$
d) $4x + 3 = -9$
e) $\frac{1}{6}x + \frac{1}{4}x = 1$
f) $9(x - 1) = 6(x + 2) - x$
g) $2n - \frac{3}{4}n = \frac{1}{2}n + \frac{13}{3}$
h) $3(A + 2) = 2(A - 7)$
i) $4(a - 3) = -2(a - 6) + 6a$ ***
j) $4(a + 3) = -2(a - 6) + 6a$ ***
k) $\frac{1}{3}(7 - 7x) = 21$
l) $\frac{3}{4}(d - 8) = \frac{2}{3}(d + 1)$
m) $-\frac{3}{5}(10x + 25) = \frac{4}{7}(14x + 49)$
n) $-\frac{2}{3}(-x + 9) = -\frac{3}{2}(4x - 2)$

§2.3 - Percents

The key thing we need to do in the math for percents is to change percentages to decimals. Whenever we are given percents, we move the decimal over in the percentage by two spaces. Some examples are

$$95\% = .95$$
$$2\% = 2.0\% = .02$$
$$.01\% = .0001$$

Now here are some word problems so that we can see how to use percents in some applications:

Example 0.5. *A* 30% *discount on a 1 year membership for a fitness center amounted to a* 90% *savings. Find the cost of the membership before the discount.*

Example 0.6. *A real estate agent earned* \$14,025 *for sell a house. If she received a* $5\frac{1}{2}\%$ *commission, how much did the house sell for?*

Example 0.7. *A genealogist determined that worldwide, 180 out of 10 million people had the same last name as his. What percentage is this?*

Example 0.8. *An art gallery agreed to sell an artist's painting for a commission of* 45%. *What must the selling price of the painting be if the gallery wants to make* \$13,500*?*

Example 0.9. *Joe invests* \$5000 *at a rate of* 7.5% *per year. How much does he have at 6 months?*

Example 0.10. *Kevin invests* \$10000 *at the start of 2013, and at the end of the year, he has* \$12,500. *What is the percentage change in his investment?*

Example 0.11. *A population has 1 million infected individuals at the start of 2013, and at the end of the year, the population has 500,000 infected individuals. What is the percentage change in the number of infected individuals in the population?*

Practice Problems

1) Solve for x: $-\frac{3}{5}(10x+25)=\frac{4}{7}(14x+49)$

2) Solve for x: $-\frac{2}{3}(-x+9)=-\frac{3}{2}(4x-2)$

§2.4 - **Formulas**

Special Formulas to know

$r=c+m$	Retail Price
$p=r-c$	Profit
$I=Prt$	Interest
$d=rt$	Distance
$P=2l+2w$	Perimeter of rectangle
$A=\pi r^2$	Area of circle
$V=\pi r^2 h$	Volume of a cylinder
$V=\frac{1}{3}\pi r^2 h$	Volume of a cone
$V=\frac{4}{3}\pi r^3$	Volume of a sphere

Solve for y or the indicated variable:

Example 0.12. *a)* $3x+4y=1$
b) $-2x-5y=10$
c) Solve perimeter formula for w
d) $\frac{3}{4}x+\frac{1}{2}y=6$
e) $\frac{1}{6}x+\frac{1}{4}y=1$
f) $S=2\pi(r^2+rh)$ *Solve for* h *(Challenge: solve for* r*)*
g) $K=\frac{1}{2}mv^2$ *Solve for* v
h) $E=mc^2$ *Solve for* c
i) $T=2r+2t$ *Solve for* r
j) $3(x-1)-2(y-6)=6$
k) $2(x+3)-3(y-2)=4(x-2)$

Practice Problems

1) Solve for h: $S=\frac{ph}{2}+B$

2) Solve for h: $S=2\pi(r^2+rh)$

3) Solve for x: $y-y_1=m(x-x_1)$

§2.5 - **Problem Solving**

Example 0.13. *A trucking company had its logo embroidered on the front of baseball caps. It was charged* \$8.90 *per hat plus a one time fee of* \$25*. If the project cost* \$559*, how many hats were embroidered?*

Example 0.14. *A classic car owner is going to sell his 1959 Chevy Impala at auction. He wants to make* \$46,000 *after paying an* 8% *commission to the auctioneer What should the selling price of the car be?*

Example 0.15. *The year George Washington was chosen president and the year the Bill of Rights went into effect are consecutive odd integers whose sum is 3580. Find the years.*

Example 0.16. *Police used 400 feet of yellow tape to fence off a rectangular lot for an investigation. They used 50 fewer feet of tape for each width as they did for each length. Find the dimensions of the lot.*

Example 0.17. *If the vertex angle of an isosceles triangle is 56 degrees, find the measure of each base angle.*

Example 0.18. *It costs Bill* \$65 *per day plus 25 cents per mile to rent a car. If the car is rented for two days, how many miles can Bill drive on a* \$275 *budget?*

§2.6 - **More Problem Solving**

Example 0.19. *A college student wants ton invest the* \$12,000 *inheritance he received and use the annual interest earned to pay his tuition cost of* \$945. *The highest rate offered by the bank is* 6% *interest. At this rate, he cannot earn the needed* \$945*, so he decides to invest in a riskier, more profitable investment offering* 9% *return. How much should he invest at each rate?*

Example 0.20. *A cargo ship, heading into port, radios the Coast Guard that it is experiencing engine trouble and that its speed has dropped to 3 mph. Immediately the Coasts Guard ship leaves and speeds at 25 mph towards the disabled ship which is 56 miles away. How long will it take for the Coast Guard ship to reach the boat?*

Example 0.21. *While on tour, a country music star travels by bus. Her musical equipment is carried in a truck. How long will it take for her bus traveling at 60 mph to overtake the truck traveling at 45 mph, if the truck had a 1.5 hour head start to her next concert?*

Example 0.22. *A chemistry experiment calls for a* 30% *sulfuric acid solution. If the lab room only has* 50% *and* 20% *solution, how much of each is needed in order to make 12 liters of* 30% *solution?*

Example 0.23. *Colombian coffee costs 9 dollars per pound and French coffee costs 6 dollars per pound. How many pounds of French should be mixed with 50 pounds of Colombian to make a mixture that costs 7 dollars per pound?*

Example 0.24. *A restaurant owner needs to purchase some tables, chairs, and plates for the dining area. She plans on buying 4 chairs and 4 plates for each table. She also plans on buying an additional 20 plates in case of a breakage. If a table costs 100 dollars, chairs cost 50 dollars, and plates cost 5 dollars. How many of each item can she buy for 6500 dollars?*

Example 0.25. *A cyclist leaves his training base for a morning workout riding at 18 mph. Then 1.5 hours later, his support stagg leaves at 45 mph in the same direction. How long will it take for the the support staff to catch up to the cyclist?*

§2.7 - **Solving Inequalities**

A linear inequality in one variable can be written as one of the following:

$$\begin{cases} ax+b>c \\ ax+b\geq c \\ ax+b<c \\ ax+b\leq c \end{cases}$$

We will also need to know what interval notation is. We know that a closed dot, $\cdot$, is used for the $\leq, \geq$ symbols and that the $\circ$ is used for the $>, <$ symbols. For interval notation, when we graph our solution on the number line, the symbols], [(closed brackets) are used for a closed dot (depending on the side), and), ((open brackets) are used for the open dot. A special situation is when we have $\pm\infty$. In this case, they always have the open brackets. So we would see $+\infty)$ or $(-\infty$. For example, the solution set of $x \geq 0$ would be written as $[0, \infty)$. Similarly $x < 4$ would be $(-\infty, 4)$.

Example 0.26. *Is 9 a solution of* $2x + 4 \leq 21$*? What about* $2x + 4 > 21$*?*

Example 0.27. *Graph on a number line* $x \leq 2$*.*

Example 0.28. *Graph on a number line* $-4 \leq x < 0$*.*

Example 0.29. *Solve* $x + 3 > 2$ *and write the solution in interval notation.*

For the following, solve the inequality, write the solution in interval notation, and graph it.

Example 0.30. $-\frac{3}{2}t \geq -12$

Example 0.31. $-5t \geq 55$

Example 0.32. $-5 > 3x + 7$

Example 0.33. $8(y + 1) \geq 2(y - 4) + y$

Example 0.34. $5(b - 2) \geq -(b - 3) + 2b$

Example 0.35. $\frac{3}{4} + \frac{x}{2} > \frac{6}{7}$

Example 0.36. $-4 < 2(x - 1) \leq 4$

Example 0.37. $x > 5$ *and* $x \leq 3$