MATH 65B **FINAL** **July 27, 2016**

Name: ______________________________ **Score:** ________ / 100

Student ID: ______________________________

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	Total
✓										70
Score										
Pts. Possible	6	8	8	8	8	8	12	12	12	82

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 9 question exam.
- Students have 2 hours and 15 minutes to complete the exam.
- The test will be out of **70 points**. The highest possible score will be **82 points**. You can attempt as many of the questions as you wish, but keep in mind you are trying to get to the **70 points**.
- In the above table, the row with the ✓, is for you to keep track of the problems you are attempting/completing.
- Higher point problems are harder, thus they are weighted more. In order to do well, you will have to attempt some of the more difficult problems.
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. This means you need to state which test you are using for series questions! Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The back of the test can be used for scratch work.

GOOD LUCK!

FORMULAS:

Common Taylor Series	Common Taylor Series
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad \text{for all } \lvert x\rvert < 1$	$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \quad \text{for all } x \in \mathbb{R}$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \text{for all } x \in \mathbb{R}$	$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \quad \text{for all } x \in \mathbb{R}$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}, \quad \text{for } x \in (-1, 1]$	$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \quad \text{for } \lvert x\rvert \leq 1$
$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n, \quad \text{for } \lvert x-a\rvert < R$	$(1+x)^m = \sum_{n=0}^{\infty} \binom{m}{n} x^n, \quad \text{for } \lvert x\rvert < 1$

1) (3 pts.) (a) Determine whether the sequence converges or diverges:

$$a_n = \frac{\cos^2(n)}{2^n}$$

(3 pts.) (b) Determine whether the series converges or diverges. If it converges, find its sum:

$$\sum_{n=1}^{\infty} \arctan(n)$$

2) (6 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$$

3) (8 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$$

4) (4 pts.) (a) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$$

(4 pts.) (b) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}$$

5) (8 pts.) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^2-1}}$$

6) (8 pts.) Find the radius of convergence and interval of convergence for the following power series (This is known as the *Bessel function of order 1*):

$$\sum_{n=0}^{\infty} \frac{(-1)^n \, x^{2n+1}}{n! \, (n+1)! \, 2^{2n+1}}$$

7) (6 pts.) (a) Compute the following integral using Taylor series. State the radius of convergence. (*Hint: Be careful about the* $n = 0$ *term, you can't have 0/0.*)

$$\int \frac{e^x}{x}\, dx$$

(6 pts.) (b) Find the Taylor series centered at $a = 0$, and state the radius of convergence.

$$f(x) = \ln(x)$$

8) (6 pts.) (a) Eliminate the parameter in the for the following parametric equation

$$x = 4\cos(t) + 2 \quad y = 2\sin(t) + 1$$

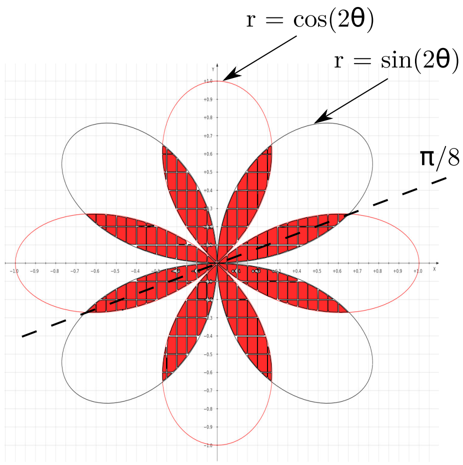
(2 pts.) (b) Identify the type of graph from your result in part (a).

(4 pts.) (c) Compute $\dfrac{dy}{dx}$. What coordinates have a tangent line with slope 0?

9) (12 pts.) Find the area of the region that lies inside the polar roses (the region is shaded in the labeled plot below).

Hint 1: Use the symmetry at $\frac{\pi}{8}$ *to simplify the integral. How many pieces do you really have?*
Hint 2: Use the identity: $\sin^2(2\theta) = \frac{1}{2} - \frac{1}{2}\cos(4\theta)$.

$$r = \cos(2\theta)$$
$$r = \sin(2\theta)$$



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END OF TEST