Chapter 5 - Review

Topics

* Fundamental Theorem of Calculus (Parts 1 and 2)
* Indefinite Integrals
* Substitution
* Area between Curves

Fundamental Theorem of Calculus (Section 5.4)

Part 1: Let f be continuous on $[a,b]$, then $F(x) = \int_a^x f(t)\,dt$ is continuous on $[a,b]$ and differentiable on (a,b) s.t.

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Part 2: If f is continuous over $[a,b]$ and F is the antiderivative of f on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Examples

1. $\frac{d}{dx}\int_0^{\sqrt{x}} \cos(t)\,dt$

2. $\frac{d}{dx}\int_0^{x^4} \sqrt{t}\,dt$

3. $\frac{d}{dx}\int_1^x \frac{1}{t}\,dt$ for $x > 0$

4. $\frac{d}{dx}\int_{\tan(x)}^0 \frac{1}{1+t^2}\,dt$

Worked Example

Find $F'(x)$ where $F(x) = \int_0^{4x^3} e^{t^3}\,dt$

Solution: Let $G(u) = \int_0^u e^{t^3}\,dt$ (Replace $4x^3$ by u $\Rightarrow u = 4x^3$)

by FTC $\Rightarrow G'(u) = e^{u^3}$ ($'$ denoted $\frac{d}{du}$ here)

We can write $F(x) = G(u) = G(u(x))$. By Chain Rule,

$$F'(x) = \frac{d}{dx} G(u(x)) = \frac{dG(u(x))}{dx} \cdot u'(x)$$

$u'(x) = \frac{du}{dx}$

$u = 4x^3$

$\Rightarrow \frac{du}{dx} = 12x^2$

$$= e^{u^3} \cdot u'(x)$$

$$= e^{(4x^3)^3} \cdot (12x^2)$$

$$= \boxed{12x^2 e^{64x^9}}$$

Integration

Power Rule: $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$

Substitution: $\int f(g(x)) \cdot g'(x)\,dx = \int f(u)\,du$ (we "undo" chain rule)

Substitution Definite Integrals: $\int_a^b f(g(x)) \cdot g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$ (change bounds of integration)

Area between Curves:

wrt x: $A = \int_a^b [f(x) - g(x)]\,dx$

wrt y: $A = \int_c^d [f(y) - g(y)]\,dy$

for $f(x) \geq g(x)$ on $[a,b]$ (for y on $[c,d]$)

Examples

1. $\int 5\sec^2(5x+1)\,dx$
2. $\int x^2\cos(x^3)\,dx$
3. $\int x\sqrt{2x+1}\,dx$
4. $\int \frac{4x^3}{(x^4+1)^2}\,dx$
5. $\int \frac{x}{\sqrt{1+x}}\,dx$

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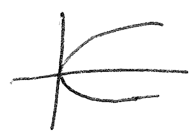
1. $\int_0^1 t^3(t^4+1)^3\,dt$
2. $\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\,dx$
3. $\int_0^1 \frac{3r}{(r^2+4)^2}\,dr$
4. $\int_0^1 t\sqrt{4+5t}\,dt$

Areas

1. $y = x^2 - 2$
   $y = 2$
2. $y = x^2$
   $y = -x^2 + 4x$
3. $x = 2y^2$
   $x = 0$
   $y = 3$



4. $y = 2\sin x$
   $y = \sin(2x)$ $\quad 0 \le x \le \pi$