Section 8.2 - Integration by Parts

We want to be able to integrate things like

$$\int f(x)g(x)\,dx$$

We essentially want to "undo" product rule

Proof: Product Rule $\Rightarrow$ $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

$$\Rightarrow \int \frac{d}{dx}[f(x)g(x)]\,dx = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

$$\Rightarrow \int f(x)g'(x)\,dx = \int \frac{d}{dx}[f(x)g(x)]\,dx - \int f'(x)g(x)\,dx$$

Letting $u = f(x)$ $v = g(x)$

$du = f'(x)\,dx$ $dv = g'(x)\,dx$

$$\Rightarrow \boxed{\int u\,dv = uv - \int v\,du}$$ Formula ⓐ

Examples

$\int x\cos(x)\,dx$ $\int x^2 e^x\,dx$ $\int_0^4 xe^{-x}$

$\int \ln(x)\,dx$ $\int e^x\cos(x)\,dx$

$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$ $f(x) = x^3$

Helpful acronym to choose u

Logarithmic
Inverse Trigonometric
Algebraic (polynomials)
Trig
Exponential

Applications

① A particle moves in a straight line with velocity $v(t) = t^2 e^{-t}$ m/s after t seconds

How far does it travel in t seconds

Average value of function $\Rightarrow f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

② A damping force slows the motion of a spring-mass system. The position is given by

$$y(t) = 2e^{-t}\cos(t) \quad t \geq 0$$

Find the average value for $0 \leq t \leq 2\pi$