Section 10.3 - Integral Test

Integral Test: Suppose f is continuous, positive and decreasing on $[1, \infty)$ and let $a_n = f(n)$. then

1. If $\int_1^\infty f(x)\,dx$ is convergent, then $\sum_{n=1}^\infty a_n$ is convergent
2. If $\int_1^\infty f(x)\,dx$ is divergent, then $\sum_{n=1}^\infty a_n$ is divergent

Note: It is possible to start at a different integer than 1. Also, must be decreasing for ~~some~~ all x larger than some integer N.

Example: $\sum_{n=1}^\infty \frac{1}{n^2+1}$ $\left(\int_1^\infty \frac{1}{x^2+1}\,dx = \arctan(x)\Big|_1^\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}\right.$

p-series $\sum_{n=1}^\infty \frac{1}{n^p}$ is convergent for $p > 1$
divergent for $p \leq 1$

Example: Determine if $\sum_{n=1}^\infty \frac{\ln(n)}{n}$ conv. or div.

Examples: $\sum_{n=1}^\infty ne^{-n^2}$, $\sum_{n=1}^\infty \frac{1}{2^{\ln(n)}}$