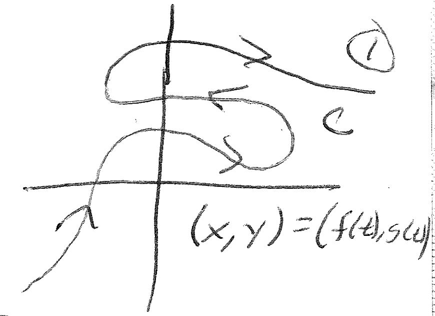
Section 11.1 - Parametric Equations



Imagine a particle moves along a curve C in the figure. We cannot describe C by a function $f(x) = y$ as it is not a function.

If x and y are functions of a 3^{rd} variable, t, called the parameters, we have

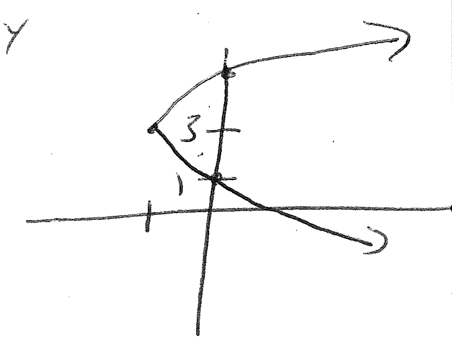
$$x = f(t) \quad \text{and} \quad y = g(t)$$

These are parametric equations. Each t yields a point (x, y) in $\mathbb{R}^2$, which can trace out C. We then call C a parametric curve.

Ex.) Sketch and identify the curve described by

$$x = t^2 - 2t$$
$$y = t + 1$$

t	x	y
-2	8	-1
-1	3	0
0	0	1
1	-1	2
2	0	3



Appears to be a parabola.
We can "eliminate the parameter"

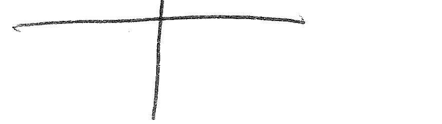
$x = t^2 - 2t$ from other, $y - 1 = t$

$$x = (y-1)^2 - 2(y-1)$$
$$x = y^2 - 4y + 3 \Rightarrow \text{parabola}$$

We can restrict values, say

$$x = t^2 - 2t \qquad 0 \le t \le 4$$
$$y = t + 1$$

$\Rightarrow$



$x = f(t)$
$y = g(t)$
$a \le t \le b$

$\Rightarrow$ $(f(a), g(a))$ = initial pt.
$(f(b), g(b))$ = terminal pt.

Ex) $x = \cos(t)$ $\Rightarrow$ $x^2+y^2 = \cos^2(t)+\sin^2(t) = 1$

$y = \sin(t)$ $\Rightarrow x^2+y^2 = 1$ $\Rightarrow$ circle

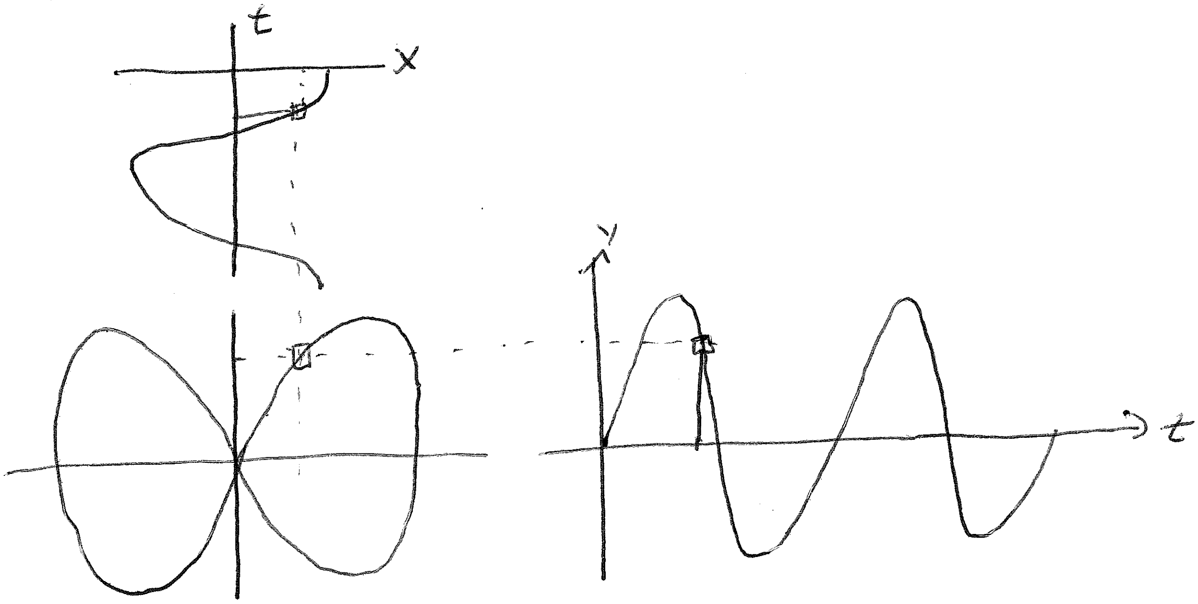
Ex) $x = h + r\cos(t)$ $\Rightarrow$ $x-h = r\cos(t)$

$y = k + r\sin(t)$ $\quad y-k = r\sin(t)$

$\Rightarrow (x-h)^2+(y-k)^2 = r^2\cos^2(t)+r^2\sin^2(t)$

$(x-h)^2+(y-k)^2 = r^2$

Ex) Sketch $x = \cos(t)$ $y = \sin(2t)$



Ex) $x = \sqrt{t}$ $t \geq 0$ $\Rightarrow y = t = (\sqrt{t})^2 = x^2$

$y = t$

Ex) $x = t$ all t

$y = t^2$

Ex) $x = t + \frac{1}{t}$ $t > 0$ find $x-y$

$y = t - \frac{1}{t}$ $\quad x+y$

$(x-y)(x+y) = x^2 - y^2 = 4$ hyperbola piece