Section 7.6 - Inverse Trig Functions

This is for the calculus part of Inverse Trig functions.

Identities

$\cos^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(x)$

$\cot^{-1}(x) = \pi/2 - \tan^{-1}(x)$

$\csc^{-1}(x) = \pi/2 - \sec^{-1}(x)$

Derivatives

$\frac{d}{dx}\sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \quad |u| < 1$

$\frac{d}{dx}\cos^{-1}(u) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \quad |u| < 1$

$\frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2}\frac{du}{dx}$

$\frac{d}{dx}\cot^{-1}(u) = -\frac{1}{1+u^2}\frac{du}{dx}$

$\frac{d}{dx}\sec^{-1}(u) = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \quad |u| > 1$

$\frac{d}{dx}\csc^{-1}(u) = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \quad |u| > 1$

Integrals

$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \quad u^2 < a^2$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\left|\frac{u}{a}\right|\right) + C \quad |u| > a > 0$

$4x - x^2 = -(x^2-4x) = -(x^2-4x+4)+4$
$= 4-(x-2)^2$

Examples

① $\int_{\sqrt{2}/2}^{\sqrt{3}/2} \frac{dx}{\sqrt{1-x^2}}$ (Ans. $= \pi/12$)

② $\int \frac{dx}{\sqrt{3-4x^2}}$

③ $\int \frac{dx}{\sqrt{4x-x^2}} = \int \frac{dx}{\sqrt{4-(x-2)^2}}$

④ $\int \frac{dx}{4x^2+4x+2}$

$4x^2+4x+2$
$= 4(x^2+x)+2$
$= 4(x^2+x+\frac{1}{4}) + 2 - \frac{4}{4}$
$= 4(x+\frac{1}{2})^2+1$
$= (2x+1)^2+1$

(5) $\int \frac{dx}{\sqrt{e^{2x}-6}}$

(6) Initial Value Problems

(1) $\frac{dy}{dx} = y$

we need to find a function $y(x)$ such that its derivative is equal to itself

$\Rightarrow \frac{1}{y}\,dy = dx \Rightarrow e^{\ln|y|} = e^{x+C}$

$y(0) = 2$

$|y| = Ce^{x}$

$\Rightarrow \boxed{y(x) = Ce^{x}}$

(2) $\frac{dy}{dx} = 1+y^2$

(3) $\frac{dy}{dx} = \frac{1}{1+x^2}$ $y(0)=1$

(4) $\frac{dy}{dx} = \sin(x)$ $y(\pi)=1$