MATH 65B **PRACTICE FINAL** **July 27, 2016**

Name: ______________________________ **Score:** ________ / 100

Student ID: ______________________________

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	Total
✓										70
Score										
Pts. Possible	6	8	8	8	8	8	12	12	12	82

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 9 question exam.
- Students have 2 hours and 15 minutes to complete the exam.
- The test will be out of **70 points**. The highest possible score will be **82 points**. You can attempt as many of the questions as you wish, but keep in mind you are trying to get to the **70 points**.
- In the above table, the row with the ✓, is for you to keep track of the problems you are attempting/completing.
- Higher point problems are harder, thus they are weighted more. In order to do well, you will have to attempt some of the more difficult problems.
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK**. Any unjustified claims will receive no credit. This means you need to state which test you are using for series questions! Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The back of the test can be used for scratch work.

GOOD LUCK!

FORMULAS:

Common Taylor Series	Common Taylor Series
$\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad \text{for all } \lvert x\rvert < 1$	$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!}, \quad \text{for all } x \in \mathbb{R}$
$e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}, \quad \text{for all } x \in \mathbb{R}$	$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!}, \quad \text{for all } x \in \mathbb{R}$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{x^n}{n}, \quad \text{for } x \in (-1, 1]$	$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1}, \quad \text{for } \lvert x\rvert \le 1$
$f(x) = \sum_{n=0}^{\infty} \dfrac{f^{(n)}(a)}{n!}(x-a)^n, \quad \text{for } \lvert x-a\rvert < R$	$(1+x)^m = \sum_{n=0}^{\infty} \binom{m}{n} x^n, \quad \text{for } \lvert x\rvert < 1$

1) (3 pts.) (a) Determine whether the sequence converges or diverges:

$$a_n = \frac{(2n-1)!}{(2n+1)!}$$

(3 pts.) (b) Determine whether the series converges or diverges. If it converges, find the sum.

$$\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$$

2) (6 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=2}^{\infty} \frac{1}{n^2+4}$$

3) (8 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$$

4) (4 pts.) (a) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$$

(4 pts.) (b) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$$

5) (8 pts.) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=2}^{\infty}(-1)^n\frac{n^n}{n!}$$

6) (8 pts.) Find the radius of convergence and interval of convergence for the following power series:

$$\sum_{n=0}^{\infty}(-1)^n\frac{(x-3)^n}{2n+1}$$

7) (6 pts.) (a) Compute the following integral using Taylor series.

$$\int \arctan(x^2)\ dx$$

(6 pts.) (b) Find the Taylor series centered at $a = \dfrac{\pi}{2}$ for

$$f(x) = \sin(x)$$

8) (6 pts.) (a) Eliminate the parameter in the for the following parametric equation

$$x = 9\cos(t) + 4 \quad y = 9\sin(t) + 1$$

(2 pts.) (b) Identify the type of graph from your result in part (a).

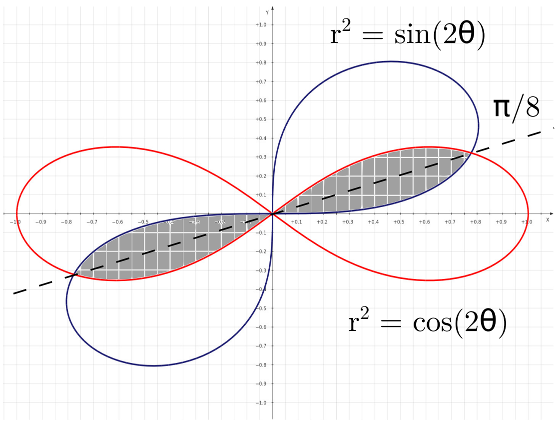
(4 pts.) (c) Compute $\frac{dy}{dx}$. What coordinates have a tangent line with slope 0? With undefined slope?

9) (12 pts.) Find the area of the region that lies inside the petals (the region is shaded in the labeled plot below).

Hint: Use the symmetry at $\frac{\pi}{8}$ to simplify the integral. How many pieces are there?

$$r^2 = \cos(2\theta)$$

$$r^2 = \sin(2\theta)$$



THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST