Last Name: First Name: Key

Math 65B - Summer 2016

Quiz 11: Tuesday July 19, 2016

1. (1 point) Find the radius of convergence and interval of convergence for the power series.

$$\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(4x+1)^{n+1}}{(n+1)^2}\cdot\frac{n^2}{(4x+1)}\right|$$

$$= \lim_{n\to\infty}\frac{n^2}{(n+1)^2}|4x+1|$$

$$= |4x+1| < 1 \quad \text{by Ratio Test}$$

$$\Rightarrow -1 < 4x+1 < 1$$

$$-2 < 4x < 0$$

$$-\frac{1}{2} < x < 0 \Rightarrow \boxed{R = 1/4}$$

Check endpoints:

$x = -1/2$

$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ which is convergent by alternating series

$\lim_{n\to\infty} \frac{1}{n^2} = 0$

$\frac{1}{(n+1)^2} < \frac{1}{n}$ for $n \geq 1$

$\Rightarrow$ decreasing

$x = 0$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2}$

convergent p-series

Interval of convergence is $[-1/2, 0]$

Please, show all work.

2. (1 point) Find the Taylor series centered at zero for the following function.

$$f(x) = x\cos\left(\frac{1}{2}x^2\right)$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\Rightarrow \cos\left(\tfrac{1}{2}x^2\right) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{2}x^2\right)^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{2^{2n}(2n)!}$$

$$\Rightarrow f(x) = x\cos\left(\tfrac{1}{2}x^2\right) = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{4n+1}}{2^{2n}(2n)!}$$

Since Taylor series for $\cos(x)$ has $R=\infty$, $x\cos(\frac{1}{2}x^2)$ also has $R=\infty$. Substitutions and multiplication by x does not change this.

Please, show all work.