LAST NAME: FIRST NAME: KEY

Math 65B - Summer 2016

Quiz 12: Thursday July 21, 2016

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1. (1 point) Eliminate the parameter for the following parameterized curve. Sketch the curve and use arrows to denote the direction.

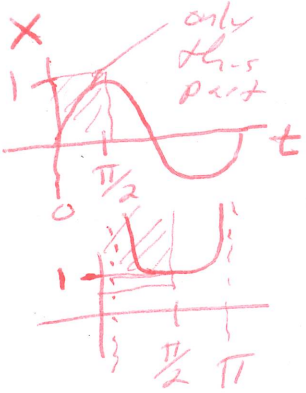
$$x = \sin(t), \quad y = \csc(t), \quad 0 < t < \frac{\pi}{2}$$

$y = \csc(t) = \dfrac{1}{\sin(t)} = \dfrac{1}{x}$

with the restriction $0 < t < \frac{\pi}{2}$

$\Rightarrow$ $0 < x < 1$ since $x = \sin(t)$

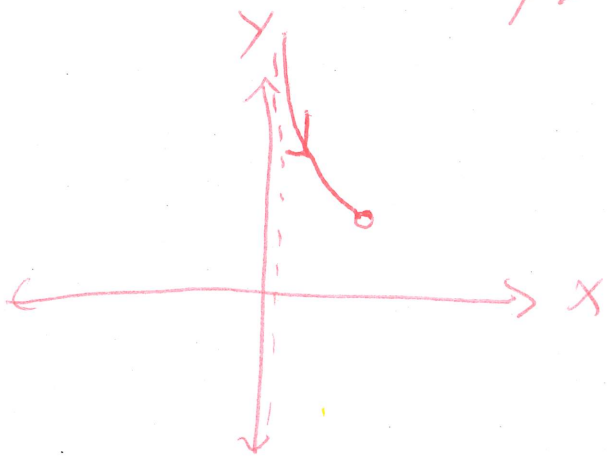
$y > 1$ since $y = \csc(t)$



So we have the graph of

$y = \dfrac{1}{x}$ for $0 < x < 1$

$y > 1$



Please, show all work.

2. (1 point) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For what values of t is the curve concave up?

$$x = 2\sin(t), \quad y = 3\cos(t), \quad 0 < t < 2\pi$$

$\frac{dy}{dt} = -3\sin(t)$ $\quad \frac{dx}{dt} = 2\cos(t)$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-3\sin(t)}{2\cos(t)} = -\frac{3}{2}\tan(t)}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dt}\right)}{dx/dt} = \frac{\frac{d}{dt}\left(-\frac{3}{2}\tan(t)\right)}{2\cos(t)} = \frac{-\frac{3}{2}\sec^2(t)}{2\cos(t)}$

$= \boxed{-\frac{3}{4}\sec^3(t)}$

Curve is concave up where $\frac{d^2y}{dx^2} > 0$

$-\frac{3}{4}\sec^3(t) > 0$

$\Rightarrow \sec^3(t) < 0$

$\Rightarrow \sec(t) < 0$

$\Rightarrow \cos(t) < 0$ on $0 < t < 2\pi$

$\Rightarrow \boxed{\text{Concave up for } \frac{\pi}{2} < t < \frac{3\pi}{2}}$

Please, show all work.