LAST NAME: FIRST NAME: KEY

Math 65B - Summer 2016

Quiz 1: Tuesday June 7, 2016

1. (1 point) Evaluate the integral using substitution

$$\int \frac{\sec^2(1/x)}{x^2}\,dx$$

Let $u = \frac{1}{x} \Rightarrow du = -\frac{1}{x^2}\,dx$

$$\Rightarrow \int \frac{\sec^2(1/x)}{x^2}\,dx = -\int \sec^2(u)\,du$$

$$= -\tan(u) + C$$

$$= \boxed{-\tan\left(\frac{1}{x}\right) + C}$$

Please, show all work.

2. (1 point) Find the area of the region bounded by $y = \cos(x), y = \sin(2x), x = 0, x = \frac{\pi}{2}$.

To find intersection points, we set two functions equal, and solve for x

$\Rightarrow \cos(x) = \sin(2x)$

$\Rightarrow \cos(x) = 2\sin(x)\cos(x)$ or $2\sin(x)\cos(x) - \cos(x) = 0$

$\Rightarrow \sin(x) = \frac{1}{2}$ and $\cos(x) = 0$

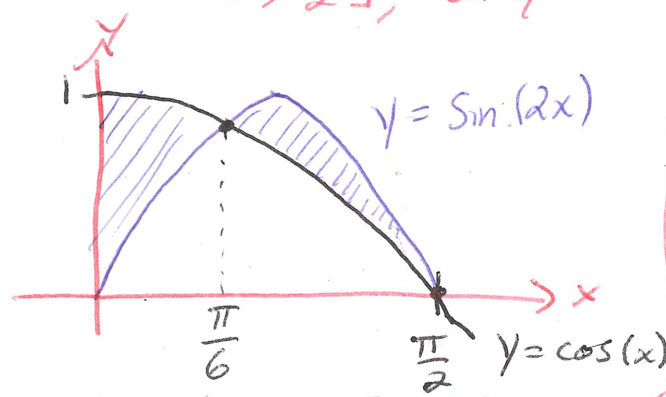
$\cos(x)(2\sin(x) - 1) = 0$

$\Rightarrow \cos(x) = 0$

$\sin(x) = \frac{1}{2}$.

We look for $x \in [0, \frac{\pi}{2}]$, only two choices, $x = \frac{\pi}{6}, \frac{\pi}{2}$

Sketch:



* Test values on $[0, \pi/6]$

$\cos(0) = 1$
$\sin(2 \cdot 0) = 0$
$\Rightarrow \cos(x) = f(x)$

Test on $[\pi/6, \pi/2]$

$\cos(\pi/4) = \sqrt{2}/2$
$\sin(2\pi/4) = 1$
$\Rightarrow \sin(x) = f(x)$

Split into two integrals, one $[0, \pi/6]$, other $[\pi/6, \pi/2]$

$$A = \int_0^{\pi/6} [\cos(x) - \sin(2x)]\,dx + \int_{\pi/6}^{\pi/2} (\sin(2x) - \cos(x))\,dx$$

$$= \left[\sin(x) + \frac{1}{2}\cos(2x)\right]\Big|_0^{\pi/6} + \left[-\frac{1}{2}\cos(2x) - \sin(x)\right]\Big|_{\pi/6}^{\pi/2}$$

$$= \left[\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} - \left(0 + \frac{1}{2}\cdot 1\right)\right] + \left[\left(\frac{1}{2} - 1\right) - \left(-\frac{1}{2}\cdot\frac{1}{2} - \frac{1}{2}\right)\right]$$

$$= \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}}$$

Please, show all work.