

LAST NAME:

FIRST NAME:

KEY

Math 65B - Summer 2016

Quiz 2: Thursday June 9, 2016

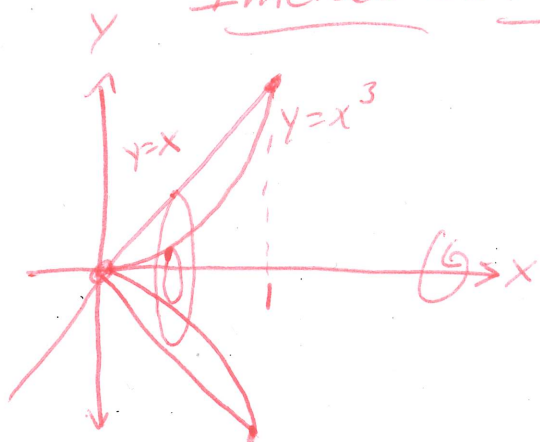
1. (1 point) Find the volume of the region bounded by $y = x^3$ and $y = x$ for $x \geq 0$, and rotated about the x -axis. (Draw the picture to help set up the problem)

Intersection Points: Solve $x^3 = x \Rightarrow x^3 - x = 0$

$$x(x^2 - 1) = 0$$

$$x = 0, \pm 1$$

$$\Rightarrow x = 0, 1 \text{ for } x \geq 0$$



Washers

$$a = 0, b = 1$$

$$R(x) = x$$

$$r(x) = x^3$$

$$\Rightarrow V = \int_a^b \pi [R(x)^2 - r(x)^2] dx$$

$$= \int_0^1 \pi [(x)^2 - (x^3)^2] dx$$

$$= \pi \left[\frac{1}{3} x^3 - \frac{1}{7} x^7 \right] \Big|_0^1$$

$$= \boxed{\frac{4}{21} \pi}$$

Please, show all work.

2. (1 point) Find the volume of the region bounded by $y = 4(x-2)^2$ and $y = x^2 - 4x + 7$, and rotated about the y-axis.

Intersection Points: $4(x-2)^2 = x^2 - 4x + 7$

$$\Rightarrow 4x^2 - 16x + 16 = x^2 - 4x + 7$$

$$\Rightarrow 3x^2 - 12x + 9 = 0$$

$$\Rightarrow 3(x^2 - 4x + 3) = 0$$

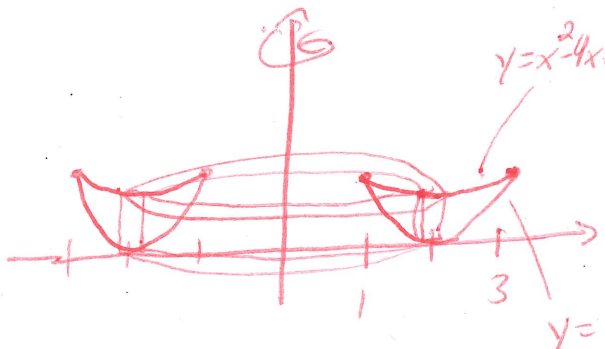
$$\Rightarrow 3(x-3)(x-1) = 0 \Rightarrow x = 1, 3$$

$$y = 4, 4$$

$$a = 1, b = 3$$

$$R(x) = x$$

$$f(x) = x^2 - 4x + 7 - (4(x-2)^2)$$



Use shell method

$$\Rightarrow V = \int_a^b 2\pi R(x) f(x) dx$$

$$\Rightarrow V = \int_1^3 2\pi (x) (x^2 - 4x + 7 - (4(x-2)^2)) dx$$

$$\Rightarrow V = 2\pi \int_1^3 x (-3x^2 + 12x - 9) dx$$

$$\Rightarrow V = -6\pi \int_1^3 (x^3 - 4x^2 + 3x) dx = -6\pi \left(-\frac{8}{3}\right)$$

$$\Rightarrow \boxed{V = 16\pi}$$

Please, show all work.