LAST NAME: FIRST NAME: KEY

Math 65B - Summer 2016

Quiz 3: Tuesday June 14, 2016

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1. (1 point) Find the arc length of the following function from $y = 0$ to $y = \frac{\pi}{4}$:

$$y = \ln(\sec(x))$$

$$y = \ln(\sec(x)) \Rightarrow \frac{dy}{dx} = \frac{1}{\sec(x)} \cdot \sec(x)\tan(x) = \tan(x)$$

$$\Rightarrow L = \int_a^b \sqrt{1+(f'(x))^2}\,dx = \int_0^{\pi/4} \sqrt{1+\tan^2(x)}\,dx$$

$$= \int_0^{\pi/4} \sqrt{\sec^2(x)}\,dx$$

$$= \int_0^{\pi/4} \sec(x)\,dx$$

$$\overset{*}{=} \ln(|\sec(x)+\tan(x)|)\Big|_0^{\pi/4}$$

$$= \ln(\sqrt{2}+1) - \ln(1)$$

$$= \boxed{\ln(\sqrt{2}+1)}$$

$(*)$ $$\int \sec(x)\,dx = \int \sec(x)\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\,dx$$

$u = \sec(x)+\tan(x)$

$du = \sec(x)\tan(x) + \sec^2(x)$

$$= \int \frac{du}{u} = \ln|u| + C$$

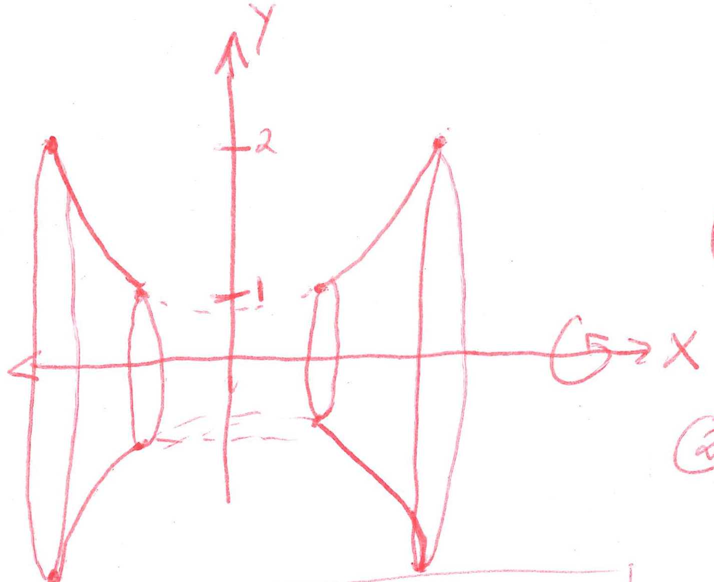
$$= \ln|\sec(x)+\tan(x)| + C$$

Please, show all work.

x-axis

2. (1 point) The given curve is rotated around the y-axis. Find the area of the resulting surface for $1 \le y \le 2$:

$$x = \frac{1}{3}(y^2+2)^{\frac{3}{2}}$$



Formulas

① $y=f(x)$ rotated around x-axis

$$S=\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

② $y=f(x)$ rotated around x-axis (alt.)

$$S=\int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

③ $x=g(y)$ rotated about x-axis

$$S=\int_c^d 2\pi y\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

④ Rotation about y-axis

$$S=\int_a^b 2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

or

$$S=\int_a^b 2\pi y\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

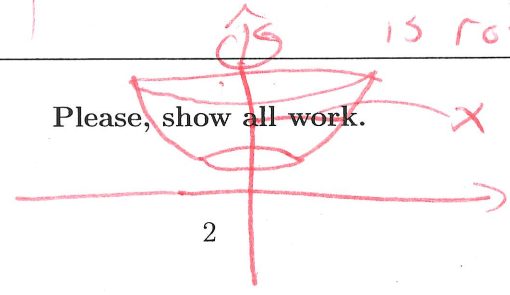
$2\pi x$ or $2\pi y$ corresponds to circumference of circle traced out by (x,y) on the curve as it is rotated.

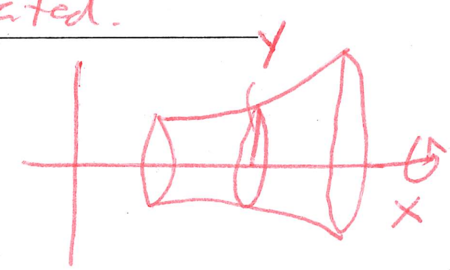
Solution

$$\frac{dx}{dy} = \frac{1}{2}(y^2+2)^{1/2}(2y)$$
$$= y\sqrt{y^2+2}$$
$$\Rightarrow 1+\left(\frac{dx}{dy}\right)^2 = 1+y^2(y^2+2)$$
$$= y^4+2y^2+1$$
$$= (y^2+1)^2$$

$\Rightarrow$ Use ③ from right

$$S=\int_1^2 2\pi\, y(y^2+1)\,dy$$
$$=2\pi\int_1^2 (y^3+y)\,dy$$
$$=2\pi\left(4+2-\tfrac{1}{4}-\tfrac{1}{2}\right)$$
$$=\boxed{\frac{21\pi}{2}}$$





Please, show all work.