

LAST NAME:

FIRST NAME:

KEY

Math 65B - Summer 2016

Quiz 4: Thursday June 16, 2016

1. (1 point) A force of 10 lb. is required to hold a spring stretched to 4 in. beyond its natural length. How much work is done in stretching it from its natural length to 6 in beyond its natural length? (Hint: Be careful with units)

We have the formula $F = kx$ or $F(x) = kx$

We need k , but have F and x , so using $4\text{in} = \frac{1}{3}\text{ft}$

$$\Rightarrow F = kx \Rightarrow k = \frac{F}{x} = \frac{10}{\frac{1}{3}} = 30 \text{ lbs/ft}$$

So then $F(x) = 30x$. The work done is (using $6\text{in} = \frac{1}{2}\text{ft}$)

$$W = \int_0^{\frac{1}{2}} 30x \, dx = 15x^2 \Big|_0^{\frac{1}{2}} = \boxed{\frac{15}{4} \text{ ft. lbs.}}$$

Please, show all work.

2. (1 point) Compute the following limit:

$$\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x$$

First compute to see what we get,

$$\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x = \left(1 + \frac{2}{\infty}\right)^\infty = (1+0)^\infty = 1^\infty$$

So we use the logarithm trick,

$$\begin{aligned} \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x &= \lim_{x \rightarrow \infty} e^{\ln\left(\left(1 + \frac{2}{x}\right)^x\right)} \\ &= e^{\lim_{x \rightarrow \infty} x \ln\left(1 + \frac{2}{x}\right)} \\ &= e^{\lim_{x \rightarrow \infty} \frac{\ln\left(1 + \frac{2}{x}\right)}{\frac{1}{x}}} \\ &= e^{\lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{2}{x}} \cdot \left(-\frac{2}{x^2}\right)}{-\frac{1}{x^2}}} \\ &= e^{\lim_{x \rightarrow \infty} \frac{2}{1 + \frac{2}{x}}} \\ &= \boxed{e^2} \end{aligned}$$

Please, show all work.