

LAST NAME:

FIRST NAME:

KEY

Math 65B - Summer 2016

Quiz 7: Tuesday June 28, 2016

1. (1 point) Evaluate the integral:

$$\int \frac{x^3}{\sqrt{x^2+9}} dx$$

$$\begin{aligned} \int \frac{x^3}{\sqrt{x^2+9}} dx &= \int \frac{3^3 \tan^3 \theta}{3 \sec \theta} 3 \sec^2 \theta d\theta \\ &= 3^3 \int \tan^3 \theta \sec \theta d\theta \\ &= 3^3 \int \tan^2 \theta \tan \theta \sec \theta d\theta \\ &= 3^3 \int (\sec^2 \theta - 1) \tan \theta \sec \theta d\theta \\ &= 3^3 \int (u^2 - 1) du \\ &= 3^3 \left(\frac{1}{3} \sec^3 \theta - \sec \theta \right) + C \end{aligned}$$

$$= 3^3 \left(\frac{1}{3} \left(\frac{\sqrt{x^2+9}}{3} \right)^3 - \frac{\sqrt{x^2+9}}{3} \right) + C$$

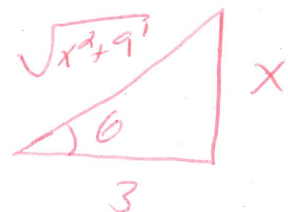
$$= \frac{1}{3} (\sqrt{x^2+9})^3 - 9\sqrt{x^2+9} + C$$

or

$$\frac{1}{3} (x^2 - 18) \sqrt{x^2+9} + C$$

$$\begin{aligned} x &= 3 \tan \theta \\ dx &= 3 \sec^2 \theta d\theta \\ \sqrt{x^2+9} &= 3 \sec \theta \end{aligned}$$

$$\begin{aligned} u &= \sec \theta \\ du &= \sec \theta \tan \theta d\theta \end{aligned}$$



$$\sec \theta = \frac{\sqrt{x^2+9}}{3}$$

Please, show all work.

2. (1 point) Evaluate the integral (Hint: Use do a substitution, then use partial fractions):

$$\int \frac{1}{x\sqrt{x+1}} dx$$

$$\text{Let } u = \sqrt{x+1} \Rightarrow x = u^2 - 1 \\ dx = 2u du$$

$$\Rightarrow \int \frac{1}{x\sqrt{x+1}} dx = \int \frac{2u du}{u(u^2-1)} = 2 \int \frac{1}{(u^2-1)} du$$

$$= 2 \int \frac{1}{(u-1)(u+1)} du = 2 \int \frac{-1}{u+1} du + 2 \int \frac{1}{u-1} du$$

$$= -\frac{2}{2} \ln|u+1| + \frac{2}{2} \ln|u-1| + C$$

$$= -\frac{2}{2} \ln|\sqrt{x+1}+1| + \frac{2}{2} \ln|\sqrt{x+1}-1| + C$$

$$= -\ln|\sqrt{x+1}+1| + \ln|\sqrt{x+1}-1| + C$$

or

$$\ln \left| \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1} \right| + C$$

$$\frac{A}{u+1} + \frac{B}{u-1} = \frac{1}{u^2-1}$$

$$A(u-1) + B(u+1) = 1$$

$$\Rightarrow B = \frac{1}{2}, A = -\frac{1}{2}$$

Please, show all work.