LAST NAME: FIRST NAME: KEY

Math 65B - Summer 2016

Quiz 8: Wednesday July 6, 2016

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1. (1 point) Evaluate the integral and state whether it is convergent or divergent:

$$\int_2^\infty \frac{1}{x\ln(x)}\,dx$$

$$\int_2^\infty \frac{1}{x\ln(x)}\,dx = \int_{\ln(2)}^\infty \frac{1}{u}\,du \qquad u = \ln(x), \quad du = \frac{1}{x}\,dx$$

$$= \lim_{t\to\infty} \ln|u| \Big|_{\ln(2)}^{t}$$

$$= \lim_{t\to\infty} \ln(t) - \ln(\ln(2))$$

$$= \infty - \ln(\ln(2))$$

$$= \infty$$

$$\Rightarrow \boxed{\text{divergent}}$$

Please, show all work.

2. (1 point) Determine whether the sequence converges or diverges. If it converges, find its limit.

$$(a) \qquad a_n = n^2 e^{-n}$$

$$(b) \qquad a_n = n \sin\left(\frac{1}{n}\right)$$

$$(c) \qquad a_n = \frac{n!}{2^n}$$

a) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} n^2 e^{-n} = \lim_{n\to\infty} \frac{n^2}{e^n}$

$= \lim_{x\to\infty} \frac{x^2}{e^x}$

$\overset{\text{L'H 2 times}}{=} \lim_{x\to\infty} \frac{1}{e^x}$

$= 0$

$\Rightarrow \boxed{\lim_{n\to\infty} n^2 e^{-n} = 0 \text{ convergent}}$

b) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} n \sin(1/n) = \lim_{n\to\infty} \frac{\sin(1/n)}{1/n}$ ($\frac{0}{0}$ type limit)

$u = \frac{1}{n}$ $\Rightarrow \lim_{n\to\infty} \frac{1}{n} = 0$

(or use L'Hopitals here) $= \lim_{u\to 0} \frac{\sin(u)}{u}$

$= \boxed{1 \text{ convergent}}$

c) Method 1: $\frac{n!}{2^n} = \frac{1}{2} \cdot \underbrace{\frac{2}{2} \cdot \frac{3}{2} \cdot \frac{4}{2} \cdots \frac{(n-1)}{2}}_{>1} \cdot \frac{n}{2} > \frac{1}{2} \cdot \frac{n}{2} = \frac{n}{4}$

$\Rightarrow \lim_{n\to\infty} \frac{n!}{2^n} > \lim_{n\to\infty} \frac{n}{4} = \infty \Rightarrow a_n = \frac{n!}{2^n}$ $\boxed{\text{divergent}}$

Method 2: By Thm 6, $\lim_{n\to\infty} \frac{x^n}{n!} = 0$ for all x

Let $x = 2 \Rightarrow \lim_{n\to\infty} \frac{2^n}{n!} = 0 \Rightarrow \lim_{n\to\infty} \frac{n!}{2^n} = \frac{1}{0} = \infty$

$\boxed{\text{divergent}}$