LAST NAME: FIRST NAME: KEY

Math 65B - Summer 2016

Quiz 9: Tuesday July 12, 2016

1. (1 point) Determine whether the series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty}\left(\frac{1}{e^n}+\frac{1}{n(n+1)}\right)$$

$\sum_{n=1}^{\infty} \frac{1}{e^n} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n \Rightarrow$ geometric $\frac{1}{e} < 1$ convergent

$a = \frac{1}{e}$ $r = \frac{1}{e}$ $\Rightarrow \frac{a}{1-r} = \frac{1/e}{1-1/e} = \frac{1}{e-1}$

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is convergent by lecture and $S = 1$

$\Rightarrow \sum_{n=1}^{\infty}\left(\frac{1}{e^n}+\frac{1}{n(n+1)}\right) = \sum_{n=1}^{\infty}\frac{1}{e^n} + \sum_{n=1}^{\infty}\frac{1}{n(n+1)}$

$= \frac{1}{e-1} + 1$

$= \boxed{\frac{e}{e-1}}$ convergent

Please, show all work.

2. (1 point) Determine whether the ~~sequence~~ series converges or diverges.

$$(a) \qquad \sum_{n=1}^{\infty} ne^{-n}$$

$$(b) \qquad \sum_{n=1}^{\infty} \frac{\arctan(n)}{n^3}$$

$$(c) \qquad \sum_{n=1}^{\infty} \frac{1+5^n}{1+2^n}$$

a) Use integral test

$f(x) = xe^{-x}$ $\quad e^{-x} > 0 \ \forall x$, $\quad x > 0$ for $x > 0$ $\Rightarrow$ positive

also continuous

$f'(x) = e^{-x} + (-1)(x)e^{-x}$

$= e^{-x}(1-x)$ for $x > 1$

negative

$$\int_1^\infty xe^{-x}\,dx = \lim_{t\to\infty}\int_1^t xe^{-x}\,dx$$

$$= \lim_{t\to\infty} -e^{-x}(x+1)\Big|_1^t$$

$$= -\lim_{t\to\infty} e^{-t}(t+1) + \frac{2}{e}$$

$$= 0 + \frac{2}{e} = \frac{2}{e}$$ $\Rightarrow$ convergent

b) Comparison Test with $b_n = \frac{1}{n^3}$ $\Rightarrow$ $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^3} \le \sum_{n=}^{\infty} \frac{\pi/2}{n^3}$

convergent p-series

by comparison test, series converges

c) $$\lim_{n\to\infty} \frac{1+5^n}{1+2^n} = \lim_{n\to\infty} \frac{\frac{1}{5^n}+1}{\frac{1}{5^n}+\frac{2^n}{5^n}} \Rightarrow \infty$$

$$= \frac{0+1}{0+0} = \frac{1}{0}$$ $\Rightarrow$ divergent by divergence test