Let's Make a Deal Problem/Missing Square Problem

1) Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? Explain your answer.

<table>
<thead>
<tr>
<th>Door 1</th>
<th>Door 2</th>
<th>Door 3</th>
<th>Stay with door 1</th>
<th>Switching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>Goat</td>
<td>Goat</td>
<td>Car</td>
<td>Goat</td>
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<td>Car</td>
</tr>
</tbody>
</table>

Success is $\frac{2}{3}$ or 66% if switching. 
$\Rightarrow$ Switching is better.

2) Consider the diagram below. After rearranging the shapes, how does one explain the presence of the missing square? What can you conclude? (Hint: Look at the areas of the two regions. Then use some trigonometry to draw the conclusion.)

The area of "triangle" (A) is $A = \frac{bh}{2} = \frac{13 \times 5}{2} = 32.5$

Area of components:
(1) + (2) + (3) + (4) = $A$
$A = \frac{8.3 + 5.2 + 7.8}{2}$
$= 10 + 5 + 7 + 8 = 32$

$\Rightarrow$ Areas do not match!

If (A) is indeed a triangle, 6d || ac, so then $\Delta x = \Delta y$

So why is there a hole in (B)?

$\tan(\frac{3}{8}) = x = 0.3936$ radians
$\tan(\frac{4}{5}) = y = 0.42279$ radians

So $x \neq y$, so (A) is not a triangle.
3) Sketch the graph of the following functions: \( f(x) = \sin(x + \frac{\pi}{2}) \)

\[
f(x) = \sin(x)
\]

Adding \( \frac{\pi}{2} \) is a phase shift to the left by \( \frac{\pi}{2} \) units.

\[
\Rightarrow
\]

This is

\[ f(x) = \cos(x) \]

4) Two right triangles have side \( a \) in common. \( x \) is the size of angle \( BAC \). Find \( \tan(x) \).

\[
\text{Figure 2. Right triangle}
\]

\[
\tan(41^\circ) = \frac{a}{15} \implies a = 15 \tan(41^\circ)
\]

\[
\tan(x^\circ) = \frac{a}{10} \implies \tan(x^\circ) = \frac{15 \tan(41^\circ)}{10}
\]

\[
= \frac{3 \tan(41^\circ)}{2}
\]