These notes cover examples from the lecture on § 5.6 - Logarithms and Exponentials, and the basics of § 8.1 - Right Triangles and Trig, as well as some extra examples. These cover the most important types you are likely to see. It is strongly recommended that you work more problems similar to these in order to get good at these types of problems as they are very likely to show up on quizzes and tests.

1. § 5.6 - LOGARITHMS AND EXPONENTIALS

**Definition 1.1.** So we define the logarithm as

\[ y = \log_a(x) \iff x = a^y \]

for \( a > 0 \) and \( a \neq 1 \). We also have that if

\[ \log_a(M) = \log_a(N) \Rightarrow M = N \]

**Note:** Some notation that to be aware of:

\[ \log = \log_{10} \]

\[ \ln = \log_e \]

**Note:** The log is the inverse of the exponential function. Also, you **CANNOT** plug in negative numbers into a logarithm!

**Definition 1.2. Properties for the logarithm function** \( \log_a(x) \):

(i) \( r \log_a(M) = \log_a(M^r) \)

(ii) \( \log_a(M) + \log_a(N) = \log_a(MN) \)

(iii) \( \log_a(M) - \log_a(N) = \log_a\left(\frac{M}{N}\right) \)

Here are some basic examples that you should understand how to compute.

**Example 1.** Find the value of \( \log_4(64) \).

**Example 2.** Find the value of \( \log_3(\frac{1}{9}) \).

**Example 3.** Find the value of \( \log(100) \).

**Example 4.** Find the value of \( \ln(e^{10}) \).

Here are some basic examples that you should understand how to compute.

**Example 5.** Solve \( 2 \log_5(x) = \log_5(9) \)

**Example 6.** Solve \( \log_5(x + 6) + \log_5(x + 2) = 1 \)

**Example 7.** Solve \( \ln(x) = \ln(x + 6) - \ln(x - 4) \)

**Example 8.** Solve \( -2 \log_4(x) = \log_4(9) \)

**Example 9.** Solve \( \ln(x - 3) + \ln(x - 2) = \ln(2x + 24) \)

**Example 10.** Solve \( \log(8x) - \log(1 + \sqrt{x}) = 2 \)
2. § 8.1 - Right Triangles and Trig

**Definition 2.1.** We define the trig functions as following using the diagram above.

![Right Triangle Diagram](image)

**Figure 1.** Right Triangle

\[
\begin{align*}
\sin(\theta) &= \frac{\text{opp}}{\text{hyp}} = \frac{b}{c} \\
\cos(\theta) &= \frac{\text{adj}}{\text{hyp}} = \frac{a}{c} \\
\tan(\theta) &= \frac{\text{opp}}{\text{adj}} = \frac{b}{a} \\
\csc(\theta) &= \frac{\text{hyp}}{\text{opp}} = \frac{c}{b} \\
\sec(\theta) &= \frac{\text{hyp}}{\text{adj}} = \frac{c}{a} \\
\cot(\theta) &= \frac{\text{adj}}{\text{opp}} = \frac{a}{b}
\end{align*}
\]

**Example 11.** Find the values of all the trig functions given the triangle with sides 3, 4, 5.

**Example 12.** Find the values of all the trig functions given the triangle with sides 5, 12, 13.