These exercises cover additional examples from the course that cover the material between Midterm 1 and Midterm 2. These cover the most important types you are likely to see. It is strongly recommended that you work more problems similar to these in order to get good at these types of problems as they are very likely to show up on the midterm. Many questions can be asked that will test you on multiple concepts, such as using graph transformations to graph exponential functions. Problems are separated by section. Here, I try to follow the sample midterm and give some useful advice and additional problems.

1. **Graphing Quadratics**

   *Advice:* The best way to attack these questions is to complete the square on the parabola that has the general form $f(x) = ax^2 + bx + c$. Doing this allows you to use the vertex form, $f(x) = a(x - h)^2 + k$. This gives you the vertex $(h, k)$, which is either a max or min, and the axis of symmetry, $x = h$. From this, the graph is simple to draw.

   **Example 1.** Sketch a complete graph: $f(x) = 3x^2 + 6x + 2$
   **Example 2.** Sketch a complete graph: $f(x) = x^2 + 2x - 8$
   **Example 3.** Sketch a complete graph: $f(x) = 3x^2 + 6x + 2$
   **Example 4.** Sketch a complete graph: $f(x) = -2x^2 + 2x - 3$
   **Example 5.** Sketch a complete graph: $f(x) = 3x^2 + 6x + 2$
   **Example 6.** Sketch a complete graph: $f(x) = -2x^2 + 8x + 3$

2. **Graphing Polynomials and Rational Functions**

   *Advice: Polynomials:* Here you just need to follow the rules that I gave previously (to see more details refer to the Lecture Notes for the specific lecture):
   (a) Find the roots (zeros/x-intercepts) and y-intercepts.
   (b) Find the multiplicity of each root to see if it touches or crosses.
   (c) Figure out the end behavior of the graph.
   (d) Write the sign chart to find where the function is positive and negative.
   (e) Draw the graph.

   *Rational Functions:* These are similar, just a little more involved:
   (a) Find the roots (zeros/x-intercepts) and y-intercepts.
   (b) Find the undefined points (these give the vertical asymptotes).
   (c) Find the horizontal asymptotes.
   (d) Find the multiplicity of each root to see if it touches or crosses.
   (e) Write the sign chart to find where the function is positive and negative.
   (f) Draw the graph.

   **Example 7.** Sketch a complete graph: $f(x) = (x - 1)(x + 3)^2$
   **Example 8.** Sketch a complete graph: $f(x) = (x + 1)(x - 2)(x + 4)$
   **Example 9.** Sketch a complete graph: $f(x) = x^2(x - 3)(x + 4)$
   **Example 10.** Sketch a complete graph: $f(x) = -\frac{1}{2}(x + 4)(x - 1)^3$
   **Example 11.** Sketch a complete graph: $f(x) = 2 + \frac{1}{x}$
Example 12. Sketch a complete graph: \( f(x) = \frac{x^2 + 4}{x^2} \)

Example 13. Sketch a complete graph: \( f(x) = \frac{-1}{x^2 + 4x + 4} \)

Example 14. Sketch a complete graph: \( f(x) = 1 + \frac{1}{x-1} \)

3. **Rational Inequalities**

*Advice:* Here you just need to follow the rules that I gave in lecture:

(a) Using **ONLY** addition and subtraction, move all terms to one side of the inequality so the other side is zero. Next, reduce that side to one fraction.

(a) Find the roots (zeros) by setting the numerator equal to zero.

(b) Find the undefined points be setting the denominator equal to zero.

(c) Write the sign chart to find where the function is positive and negative.

(d) Check the original inequality (\(\leq, \geq, <, >\)).

(e) Write the answer of the correct intervals in interval notation.

Example 15. Solve: \( \frac{x+5}{x-1} > 5 \)

Example 16. Solve: \( \frac{x^2 - 2}{x-3} \geq 0 \)

Example 17. Solve: \( \frac{5}{x-3} < \frac{3}{x+1} \)

Example 18. Solve: \( \frac{x^2 + 6x + 8}{x^2 - 9} \leq 0 \)

4. **Rational Roots Theorem Examples**

*Advice:* These problems will stand out, as usually there is no possible way to factor a polynomial higher than degree 3 by just looking at it. Chances are that if the polynomial is of degree 3, 4, or 5, then using rational roots test is the way to go. Find your \(p\) and \(q\) values, and the possible rational roots. Check to see which ones are zeros by plugging them into the \(f(x)\) that is given. Then use synthetic division to reduce the polynomial into factored form. Then write out all of the roots.

Example 19. Find all zeros: \( 2x^4 + x^3 - 19x^2 - 9x + 9 \)

Example 20. Find all zeros: \( 2x^3 + 3x^2 - 8x + 3 \)

Example 21. Find all zeros: \( x^4 + x^3 - 19x^2 + 11x + 30 \)

Example 22. Find all zeros: \( x^4 - 14x^3 + 47x^2 + 14x - 48 \)

5. **Exponential Equations and Graphs**

*Advice:* **Graphs:** The best advice I can give here is that you know all of the graph transformations that we learned previously. These are extremely useful in saving time to draw the graphs. Of course this means you need to know the parent graphs, i.e.) what does \( f(x) = a^x \) look like for \( a > 1 \). What about \( 0 < a < 1 \)? If you’ve forgot the general shape, look back at the notes or plot some points to see the shapes.

**Equations:** These are all done the same way. Rewrite the terms all in the same base first. Then combine all terms into one expression on the left and right hand sides. Once this is done, you can set the exponents equal and solve for \( x \).

Example 23. Sketch a complete graph: \( f(x) = 3^{x+1} - 2 \)
Example 24. Sketch a complete graph: \( f(x) = e^{x-4} + 3 \)

Example 25. Sketch a complete graph: \( f(x) = 2^{-x} - 2 \)

Example 26. Sketch a complete graph: \( f(x) = -e^{x-1} + 1 \)

Example 27. Solve: \( 4x + 6 = 64 \)

Example 28. Solve: \( 2x^2 - 6 = 32x \)

6. LOGARITHMIC EQUATIONS

Advice: There are three types of possible problems here. All of them rely on knowing the properties of logarithm functions. First, to write an expression as a sum or difference of logarithms. Second, to write a sum or difference of logarithms as one logarithm. Third, to solve a logarithmic equation. Here I will work out an example of the first and second cases.

6.1. Log Problems: Type 1.

Example 29. Write the expression as a sum or difference of logarithms: \( \ln \left( \frac{3x^5y^3}{z^4} \right) \)

Solution: The key with these is to take them one step at a time and use the log rules wisely. You have to be able to see the “big picture”, to apply them in order. Although you can go in any order, some are easier than others. Here, use the division-subtraction rule first. Then use the product-sum rule twice. Then use the exponent rule last.

\[
\ln \left( \frac{3x^5y^3}{z^4} \right) = \ln(3x^5y^3) - \ln(z^4) \\
= \ln(3) + \ln(x^5) + \ln(y^3) - \ln(z^4) \\
= \ln(3) + 5 \ln(x) + 3 \ln(y) - 4 \ln(z)
\]

Example 30. Write the expression as a sum or difference of logarithms: \( \ln \left( \frac{6x^7y^2}{5z} \right) \)

Example 31. Write the expression as a sum or difference of logarithms: \( \log_2 \left( \frac{2x^3y^8}{z^2} \right) \)

6.2. Log Problems: Type 2.

Example 32. Write the expression as one logarithm: \( 4 \ln(x) + 5 \ln(y) - \ln(z) - 2 \ln(r) \)

Solution: This type is essentially working the Type 1 questions in reverse. The key with these is to take them one step at a time and use the log rules wisely as well. Although you can go in any order, some are easier than others. Use the exponent rule first, to get single log with no coefficients. Then work left to right, going one step at a time, this way you won’t make a mistake.

\[
4 \ln(x) + 5 \ln(y) - \ln(z) - 2 \ln(r)) = \ln(x^4) + \ln(y^5) - \ln(z) - \ln(r^2) \\
= \ln(x^4y^5) - \ln(z) - \ln(r^2) \\
= \ln \left( \frac{x^4y^5}{zr^2} \right)
\]
Example 33. Write the expression as one logarithm: $2\log_3(x) - 4\log_2(y) - \frac{1}{2}\ln(z) + 6\ln(r)$

Example 34. Write the expression as one logarithm: $\frac{1}{3}\log(x) + 3\ln(y) - 3\ln(z) - 4\ln(r)$

6.3. Log Problems: Type 3. Advice: There are plenty of example in the Lecture Notes for this section. See Lecture Notes 10 for more practice problems on these.

7. Trigonometric Graphs

Advice: Graphs: The best advice I can give here is that you know all of the graph transformations that we learned previously. These are extremely useful in saving time to draw the graphs. Of course this means you need to know the parent graphs, i.e.) what does $f(x) = \sin(x)$ look like. What about $f(x) = \cos(x)$? Also, you must remember the definitions of amplitude, period, and phase shift in order to use the transformation. These things tell you how to shift the graphs appropriately.

Example 35. Sketch a complete period: $y = 4\sin\left(x - \frac{\pi}{4}\right)$

Example 36. Sketch a complete period: $y = 2\cos\left(2\left(x - \frac{\pi}{4}\right)\right)$

Example 37. Sketch a complete period: $y = 3\sin\left(x - \frac{\pi}{2}\right) + 1$