1) Sketch a graph: \( x^2 - 4y^2 = 4 \)

*Solution:* The first step here is to rewrite the above expression:

\[
\frac{x^2}{4} - \frac{y^2}{1} = 1
\]

From this, we see that this is the form of a hyperbola, where we draw a box that goes two units to the left and right (\(x\)-direction) and 1 unit up and down (\(y\)-direction). So the vertex points are at \((2, 0)\) and \((-2, 0)\), and we can draw two lines to know where the hyperbola is contained. The graph of the hyperbola is given in red and the asymptote lines are in blue.

![Figure 1](image)

2) Tickets to play are $12 for adults and $8 for children. If a total of 200 tickets were sold for a total revenue of $2,060, how many of each type of ticket were sold?

*Solution:* Let \(x\) = the number of adult tickets and \(y\) = the number of child tickets. From the story, we know that the **TOTAL** number of tickets is 200. That means in terms of our variables, \(x + y = 200\). We also know the total revenue is $2,060. So we have the equation \(12x + 8y = 2060\). So we have the following system of equations:

\[
\begin{align*}
  x + y &= 200 \\
  12x + 8y &= 2060
\end{align*}
\]

If we multiply the first equation by \(-8\) and add the two equations together, we get \(4x = 460\), which tells us that \(x = 115\), so 115 adult tickets were sold. It is easy to determine that 85 child tickets were sold since \(115 + 85 = 200\).
3) Decompose into partial fractions: \( \frac{2x+4}{(x-1)(x^2+x+1)} \)

**Solution:** The case we have here is a linear factor and an irreducible quadratic. Using the rules in the notes, we get

\[
\frac{2x+4}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}
\]

Finding a common denominator and setting the numerators equal, we have

\[
2x + 4 = A(x^2 + x + 1) + (Bx + C)(x - 1)
\]

Following the same method as the lecture notes, if we let \( x = 1 \), notice that

\[
2(1) + 4 = A(1^2 + 1 + 1) + (B(1) + C)(1 - 1)
\]

\[
6 = 3A
\]

\[
A = 2
\]

Now we have

\[
2x + 4 = 2(x^2 + x + 1) + (Bx + C)(x - 1)
\]

Now let \( x = 0 \), so then

\[
2(0) + 4 = 2(0^2 + 0 + 1) + (B(0) + C)(0 - 1)
\]

\[
4 = 2 - C
\]

\[
-2 = C
\]

And finally, if we let \( x = -1 \) (anything except 0 and 1 will work), we get

\[
2(-1) + 4 = 2((-1)^2 - 1 + 1) + (B(-1) - 2)(-1 - 1)
\]

\[
2 = 2 + 2B + 4
\]

\[
-4 = 2B
\]

\[
-2 = B
\]

So then our final solution is

\[
\frac{2x+4}{(x-1)(x^2+x+1)} = \frac{2}{x-1} + \frac{-2x-2}{x^2+x+1}
\]

4) Solve:

\[
\begin{cases}
  y = x^2 + 3 \\
  2x + y = 18
\end{cases}
\]

**Solution:** Rewriting the above expressions, we get

\[
\begin{cases}
  y = x^2 + 3 \\
  y = -2x + 18
\end{cases}
\]

Setting the equations equal now, and after some algebra, we get

\[
x^2 + 3 = -2x + 18
\]

\[
x^2 + 2x - 15 = 0
\]

\[
(x - 3)(x + 5) = 0
\]

So we get that \( x = 3 \) and \( x = -5 \). Now we need the \( y \)-coordinates to write the answer in coordinate form. Plugging these values into the first equation, we see that we get \( y = 12 \) and \( y = 28 \) respectively. Therefore, our coordinates are \((3, 12)\) and \((-5, 28)\).
5) Solve by graphing:

\[
\begin{aligned}
  y &\leq -x^2 + 3 \\
  y &\geq x + 1
\end{aligned}
\]

Solution: The first thing to do here is to graph the parabola \( y = -x^2 + 3 \) and the line \( y = x + 1 \). Then the first inequality tells us that we are looking at the region below the parabola. The second inequality tells us that we are looking above the line. The two areas we have in common is the region between the parabola and the line. The graph and region are shown below.

**Figure 2.** Figure 2