Section 1.1 - Limits

Consider \( y = \frac{\sin x}{x} \). When \( x \) is near 1, where is \( y \) "close" to?

Look at graph, one can see \( y \approx \frac{\sin(1)}{1} \).

\[ \Rightarrow x \text{ "near" } 1 \Rightarrow y \text{ near } 0.84. \]

What happens when \( y = \frac{\sin x}{x} = \frac{\sin 0}{0} \rightarrow \frac{0}{0} \) ???

Ex) \( \lim_{x \to 1} f(x) = L \Rightarrow \lim_{x \to 1} \frac{\sin x}{x} \approx 0.84 \)

Do a chart to see

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \frac{\sin x}{x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>0.998</td>
</tr>
<tr>
<td>-0.01</td>
<td>0.999</td>
</tr>
<tr>
<td>-0.001</td>
<td>Undef</td>
</tr>
<tr>
<td>0</td>
<td>0.999</td>
</tr>
<tr>
<td>0.01</td>
<td>0.998</td>
</tr>
</tbody>
</table>

Ex) \( \lim_{x \to 3} \frac{x^3-x-6}{6x^2-19x+3} \)

Ex) \( f(x) = \begin{cases} x+1 & x<0 \\ -x^2+1 & x>0 \end{cases} \quad \lim_{x \to 0} f(x) = ? \)

Limits fail to exist if:

1. \( f(x) \) approaches different values on either side of \( c \)
2. \( f(x) \) grows without upper or lower bounds as \( x \to c \)
3. The function may oscillate as \( x \to c \)

Ex) \( \lim_{x \to 1} f(x) \) for \( f(x) = \begin{cases} x^2-2+3 & x \leq 1 \\ x & x>1 \end{cases} \)
\[ \lim_{x \to 1} \frac{1}{(x-1)^2} \]
\[ \lim_{x \to 0} \sin \left( \frac{1}{x} \right) \]

Limits of difference quotients

What is \( \lim_{h \to 0} \frac{f(h+1) - f(1)}{h} \) ?

\[
f(x)\]

\[
\begin{align*}
f(5) - f(1) &= \frac{20 - 10}{5 - 1} = \frac{10}{4} = 2.5 \\
\Rightarrow \text{Average velocity}
\end{align*}
\]

Now what about \( \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} \) ?

Pictorially:
Section 1.2 - \( \varepsilon \delta \) definition of Limit

**Definition:** Let \( I \) be an open interval containing \( c \), and let \( f \) be defined on \( I \), except possibly at \( c \). The limit of \( f(x) \) as \( x \) approaches \( c \) is \( L \), denoted by

\[
\lim_{{x \to c}} f(x) = L
\]

means that given any \( \varepsilon > 0 \), there exists \( \delta > 0 \) s.t.
for all \( x \neq c \) if \( |x - c| < \delta \), then \( |f(x) - L| < \varepsilon \)

**Ex:** Show \( \lim_{{x \to 2}} x^2 = 4 \)