1) Determine whether the following PDEs are separable, and if so, write the two ODEs for each PDE:

(a) \( t u_{xx} + xu_t = 0 \)
(b) \( u_{xx} + u_{xt} + u_t = 0 \)

*Hint: Assume a product solution and attempt find the two ODEs.*

**Solution:**

(a) Assume that \( u(x,t) = \phi(x)h(t) \), then

\[
\begin{align*}
t \phi''(x) h(t) + x \phi'(x) \phi(x) &= 0 \\
\frac{\phi''(x)}{x \phi(x)} + \frac{h'(t)}{th(t)} &= 0 \\
\frac{\phi''(x)}{x \phi(x)} &= -\frac{h'(t)}{th(t)} = -\lambda \\
\phi''(x) + \lambda x \phi(x) &= 0 \quad h'(t) - \lambda th(t) = 0
\end{align*}
\]

So the equation is separable.

(b) Assume that \( u(x,t) = \phi(x)h(t) \), then

\[
\begin{align*}
\phi''(x) h(t) + \phi'(x) h'(t) + \phi(x) h'(t) &= 0 \\
\frac{\phi''(x)}{\phi'(x) + \phi(x)} + \frac{h'(t)}{h(t)} &= 0 \\
\frac{\phi''(x)}{\phi'(x) + \phi(x)} + \frac{h'(t)}{h(t)} &= -\lambda \\
\phi''(x) + \lambda (\phi'(x) + \phi(x)) &= 0 \quad h'(t) - \lambda th(t) = 0
\end{align*}
\]

So the equation is separable. \( \square \)