Quiz 4

Name: ________________________________
Date: ________________________________

Score: ____________

Directions: Show your work for each problem and box your answers. You may not use any electronic devices, notes, or textbooks.

Problem 1. (3 points) Solve and write interval notation for the solution set. Then graph the solution set.

\[ |2x + 3| \leq 9. \]

Solution:

Recall an inequality involving the absolute value of the form

\[ |\text{expression}| \leq a \]

can be rewritten as two inequalities:

\[ \text{expression} \leq a, \text{ and } \text{expression} \geq -a. \]

So we have

\[ 2x + 3 \leq 9 \text{ and } 2x + 3 \geq -9. \]

We end up getting \(-6 \leq x \leq 3\), so the final answer is \([-6, 3]\).

Problem 2. (3 points) Find the zeros of the polynomial function and state the multiplicity of each zero

\[ f(x) = (x + 3)^2(x - 1). \]

Solution:

We set \( f(x) = 0 \) and solve for \( x \). So \( 0 = (x + 3)^2(x - 1) \) means that \( 0 = (x + 3)^2 \) and \( 0 = (x - 1) \). Hence \( x = -3 \) and \( x = 1 \), with multiplicity \( 2 \) and \( 1 \) respectively.

Problem 3. (3 points) Graph the polynomial function

\[ f(x) = -x(x - 1)^2(x + 4)^2. \]

Solution:

First, we see that the leading term of the polynomial is \(-x^5\), the end behavior will look like

Next we find the zeros of \( f(x) \) and their multiplicity. We have \( x = 0 \) with multiplicity 1 (odd, crosses), \( x = 1 \) with multiplicity 2 (even, bounce), and \( x = -4 \) with multiplicity 2 (even, bounce). Hence we draw the picture
Problem 4 (3 points) Using synthetic division, determine whether the given numbers are zeros of the
given polynomial function

\[ f(x) = x^3 - \frac{7}{2}x^2 + x - \frac{3}{2}, \text{ and } x = -3, x = \frac{1}{2} \]

Solution:

For \( x = -3 \), the remainder is \(-63\), so \( x = -3 \) is not a root. For \( x = \frac{1}{2} \), the remainder is \(-7/4\) so
\( x = -7/4 \). Hence both numbers are not roots.

Problem 5. (3 points) Find the rational zeros and then all other zeros. Then factor \( f(x) \) into linear
factors:

\[ f(x) = 3x^3 - x^2 - 15x + 5 \]

Solution:

Use the rational roots theorem. The possible roots are \( \pm 1, \pm 5, \pm \frac{1}{3}, \pm \frac{5}{3} \). Try them all using synthetic
division. We see that \( \frac{1}{3} \) works, so we can rewrite

\[ f(x) = (x - 1/3)(3x^2 - 15) \]

\[ = 3(x - 1/3)(x^2 - 5) \]

\[ = 3(x - 1/3)(x - \sqrt{5})(x + \sqrt{5}). \]

Thus we have factored \( f \) into a product of linear terms.