Freeness of Hopf Algebras

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Abstract

In 1975 Kaplansky discussed "Ten conjectures on Hopf algebras" during a lecture at the University of Chicago. The first of these conjectures concerned freeness of a Hopf algebra as a module over a subHopfalgebra. Specifically he conjectured that "a Hopf algebra is free as a module over any subHopfalgebra". Although this was quickly shown to be false in the infinite dimensional case, the finite dimensional case turned out to be true, and was proven 14 years later by Nichols and Zoeller. This result is the heart of this paper.

The Nichols-Zoeller freeness theorem states that a finite dimensional Hopf algebra is free as a module over any subHopfalgebra. We will prove this theorem, as well as the first significant generalization of this theorem, which was proven three years later. This generalization says that if the Hopf algebra is infinite dimensional, then the Hopf algebra is still free if the subHopfalgebra is finite dimensional and semisimple. We will also look at several other significant generalizations that have since been proven.