Studying for the Final Exam
MATH-10a – Vector Calculus
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As stated in the course syllabus, at least 50% of the problems on the final will come directly from the homework. So, the most obviously important thing you can do to study for the final exam is to review the homework problems and make sure you know how to do all of those. Hopefully you have already started doing this. Since some of the homework problems have several parts, I may break such problems up and only include one part as the exam question. Also, I may change the wording only slightly if it is necessary to clarify the context (e.g. if the problem said "Prove Theorem 42" I'd fill in some verbal description of Theorem 42). Other than that, the problems I pull from the homework will be *verbatim*.

The next most obvious things to study are the 5 weekly quizzes. While there is no promise that anything from these quizzes will reappear on the final exam, you should certainly make sure you know how to do those problems. Also, you have no real excuse *not* to know how to do them, since the solutions are all online.

There will be problems on the exam of a variety of difficulty levels — a few easy ones, a couple of fairly challenging ones, with most falling somewhere in the middle. Because of this, my suggested strategy for the exam (actually, for *any* math exam) is to look over all the questions and then quickly do to the easier-looking ones first, moving on to harder problems as time progresses.

Here are some problems. I make no guarantee that the problems on the final will look like these, or that these cover everything you need to know, but they are good review questions and you should know how to do them.

1. Draw the graph of $f(x,y) = x^2 - y^2$.
2. Sketch the level curves of $f(x,y) = x^2 + 9y^2 - 1$. Be sure to draw an accurate picture of the level curve where $f = 0$. Compute and draw (on the same sketch) the vector field $\vec{\nabla} f$.
3. Find all of the critical points of the function $\sin(x^2 + y^2)$. Plot some of them (in the xy plane). What type of critical point is the one at $(0,0)$?
4. If a plane is flying along the path $\mathbf{c}(t) = (t, t^4, 3)$, what is the *speed* of the plane at time $t = 2$? What is its acceleration?
5. In the previous problem, suppose that the temperature of the air is $T(x,y,z) = 75 - 2z + 3x - y$. Use the chain rule to find the rate of change of temperature along the path of the plane.
6. Compute the derivative matrix for $f(x,y) = (xy^2, 2x, 1, sin(xy))$. How do you know that f is differentiable?
7. Calculate the mixed second-partial derivatives $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ for the function $f(x,y) = \tan(xy^2 + x)$ and check that they are equal.
8. Does the function $f(x,y) = \frac{xy}{x^2+y^2}$ have a limit at $(0,0)$? [Hint: try approaching along some straight-line paths as we did in the handout homework on limits.]
9. Calculate the second order Taylor approximation of $f(x,y) = e^{x^2+y-1}$ at the point $(x_0, y_0) = (1,0)$.
10. Let $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be the vector field given by $\vec{F}(x,y,z) = (2x^2, 3x - y, z)$. Calculate the curl and divergence of $\vec{F}$.