MATH 10A
Quiz 1

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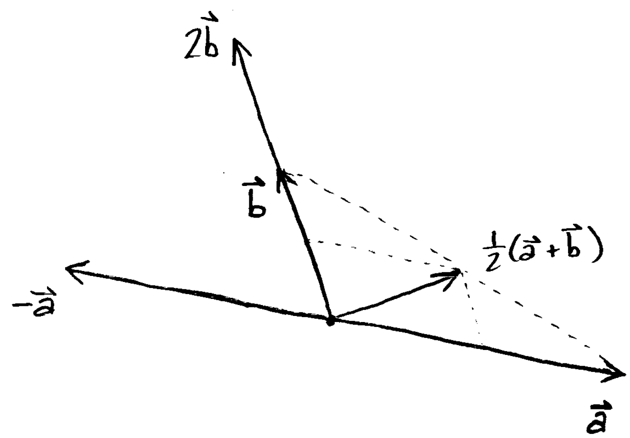
(KEY)

1.) Below is a sketch of two vectors, $\vec{a}$ & $\vec{b}$, in the plane. Draw (as accurately as you can) and label the vectors:

$-\vec{a}$

$2\vec{b}$

and $\frac{1}{2}(\vec{a}+\vec{b})$



2.) Let $\vec{v} = 3\hat{\imath} + 2\hat{\jmath} - \pi\hat{k}$ & $\vec{w} = 2\hat{\jmath} + 3\hat{k}$ be vectors in $\mathbb{R}^3$. Find:

$\vec{v}\cdot\vec{w}$, $\|\vec{v}\|$, $\|\vec{w}\|$, & the angle θ between $\vec{v}$ & $\vec{w}$

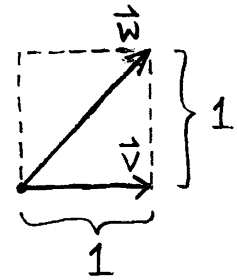
$$\vec{v}\cdot\vec{w} = 2\cdot 2 - \pi\cdot 3 = 4-3\pi$$

$$\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{3^2+2^2+(-\pi)^2} = \sqrt{13+\pi^2}$$

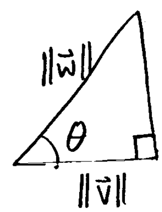
$$\|\vec{w}\| = \sqrt{\vec{w}\cdot\vec{w}} = \sqrt{2^2+3^2} = \sqrt{13}$$

$$\theta = \arccos\frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\,\|\vec{w}\|} = \arccos\frac{4-3\pi}{\sqrt{13(13+\pi^2)}}$$

3.) Suppose two vectors $\vec{v}, \vec{w} \in \mathbb{R}^3$ are oriented in such a way that $\vec{v}$ is one side of a square of length 1 on each side, and $\vec{w}$ is the diagonal:



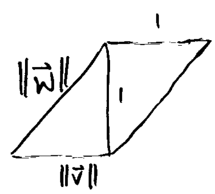
Find $\vec{v} \cdot \vec{w}$ and $\|\vec{v} \times \vec{w}\|$. Does $\vec{v} \times \vec{w}$ point down (into the page) ~~or up~~ (out of the page)? Why?



By trigonometry, $\|\vec{w}\| \cos\theta = \|\vec{v}\|$:

Therefore, $\vec{v} \cdot \vec{w} = \|\vec{v}\|\,\|\vec{w}\| \cos\theta = \|\vec{v}\|^2 = \boxed{1}$

Also, the magnitude of $\vec{v} \times \vec{w}$ is the area of the parallelogram with sides $\vec{v}$ & $\vec{w}$



So, $\|\vec{v} \times \vec{w}\| = \boxed{1}$

Finally, $\vec{v} \times \vec{w}$ points up out of the page by the right hand rule!

4.) Find $(2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}) \times (2\hat{\jmath})$

$$= 4\hat{\imath} \times \hat{\jmath} + 6\hat{\jmath} \times \hat{\jmath} - 4\hat{k} \times \hat{\jmath}$$
$$= 4\hat{k} + 6\vec{0} - 4(-\hat{\imath})$$
$$= 4(\hat{\imath} + \hat{k})$$